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Solution of a nuclear reactor parameter identification problem

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SOLUTION OF A NUCLEAR REACTOR
PARAMETER IDENTIFICATION PROBLEM

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THESIS

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Parameter Identification Problem

by

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June 1971

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ABSTRACT

A continuous identification of parameters is performed on a simulated fast breeder nuclear reactor system using hybrid computation and applying techniques of statistical regression analysis and exponentially-mapped-past functions. Output states which are not directly measurable are estimated by use of a Kalman filter.

The method developed in this study is applied to a numerical example which demonstrates that unknown parameters can be identified within 3% of their actual value, with signal noise ratios as low as 10:1 in the measured states. The example also demonstrates that convergence occurs in a reasonably short time.

An executive software routine was written in FORTRAN IV for the XDS-9300 digital computer and was applied both for the control of the process and for establishing the operational performance and accuracy of the Comcor Ci-5000 analog computer that simulated the nuclear reactor system.

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SYMBOL	DEFINITION
$C_i(t)$	Density of the i^{th} group of delayed neutron precursor at time t
$C(0)$	Density of the assumed one precursor at time t=0
$D(t)$	Normalized precursor concentration at time t
$D(0)$	Normalized precursor concentration at time t=0
$K(t)$	Reactivity function at time t
$\delta K(t)$	Reactivity input at time t
$n(t)$	Neutron density at time t
$n(0)$	Neutron density at time t = 0
$N(t)$	Normalized neutron density at time t
$N(0)$	Normalized neutron density at time t = 0
μ_j	Fraction of power developed in j^{th} medium
σ_{jk}	Thermal conductivity between the j^{th} and the k^{th} medium
ε_j	Thermal capacity of the j^{th} medium
α_j	Temperature coefficient of the reactivity for medium j
β_i	Delayed neutron fraction from i^{th} precursor
λ_i	Decay constant for the i^{th} precursor
$\rho_e(t)$	Deterministic input function of reactivity
$T_j(t)$	Variation of temperature with respect to the equilibrium temperature in medium j
ℓ^*	The prompt neutron lifetime.

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I. INTRODUCTION

Fast breeder nuclear reactors such as the recently proposed design described in [1] do not have inherent load following ability and therefore require an automatic controller to handle load changes. It is important in designing for stable operation of the automatic control system to know the value of the parameters of the plant within a reasonable degree of accuracy. These parameters change due to such factors as temperature, fuel depletion, fuel loading and power level, [2], making it desirable to have continuous identification of parameters.

The continuous identification of parameters of dynamic systems has received intensive attention during the last few years [3], [4]. References [5] and [6] among others, have shown successful results by applying gradient techniques and quasilinearization, respectively. These methods are cumbersome for nonlinear systems and in particular for dealing with a nonlinear space-dependent reactor system, [6], where the requirement of continuous identification cannot be met since the in-core sensors which are necessary for certain state measurements have not yet been developed[1].

The implementation for continuous parameter identification is difficult and complex. For example, many gradient

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[1] The procedure available consists of the exposure of a matallic foil by immersion and takes enough time (between one and two hours) to eliminate any possibility for continuous identification of parameters.

techniques, even when they represent in some cases very elegant ways of approaching the hill-climbing search procedure are limited by practical considerations in their application. To illustrate this, reference is made to a particular gradient (steepest descent) technique which uses an error criterion and sensitivity equations, and which, among other requirements, has the need for a forcing function. A block diagram of the algorithm is shown in Figure 1. In this method it is required to derive, implement and solve the sensitivity equations in order to obtain the value of the gradient for the specified cost function. In order to identify n parameters the number of analog components, using standard methods, must be multiplied by at least n * , since for a linear case these equations take the same form of the model, [7]. In addition, for nonlinear systems the sensitivity equations have more complicated forms which do not permit the use of techniques such as multiplexing, [7]. To apply these methods the system must be initialized with some previous information, usually an educated guess of the parameter values, and then the process must be iteratively reinitialized until some criterion is satisfied. Since the parameter correction is a function of the gradient and since this gradient approaches zero as the parameter value approaches the actual value (solution), the procedure will converge in a reasonable time only if a means is provided

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* More recent techniques, [8], allow for simplification, when the model can be expressed in canonical form.

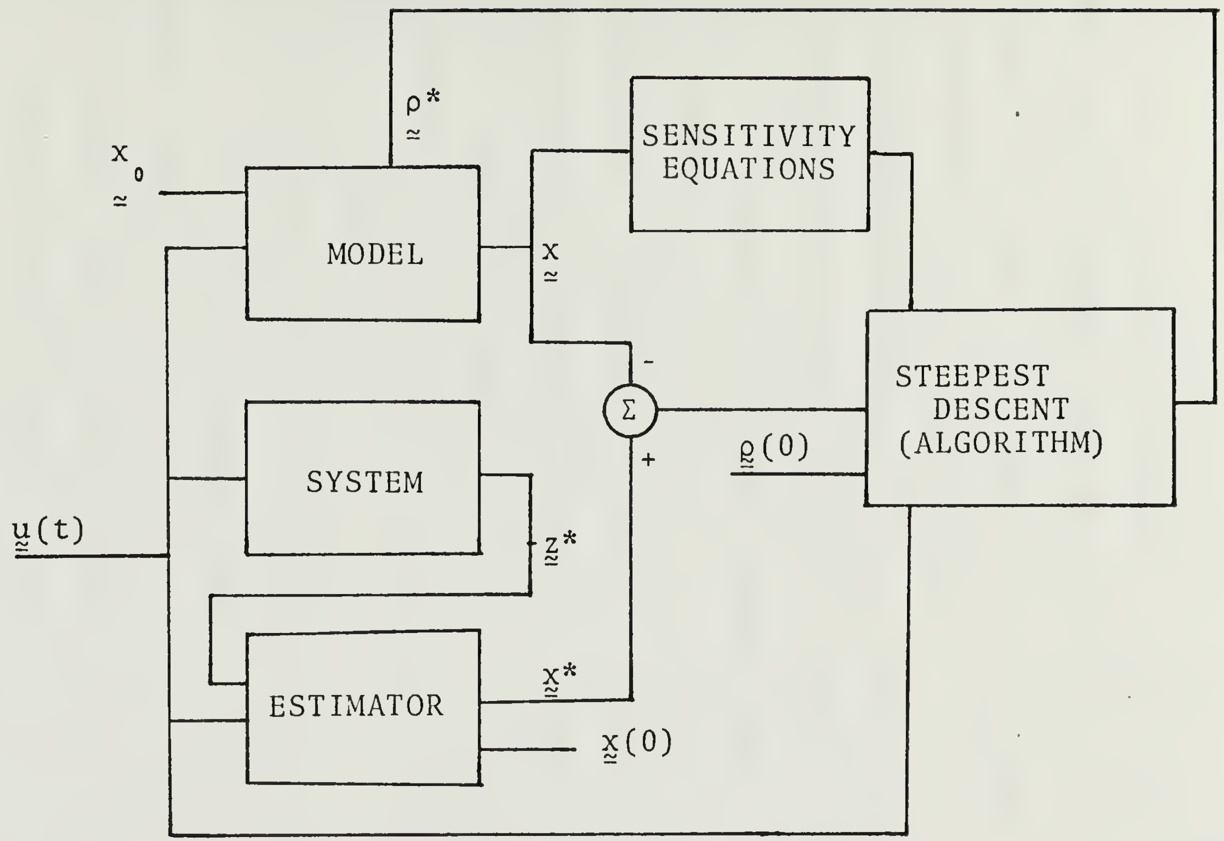

Figure 1. Identification of parameters by gradient technique using sensitivity equations.

to speed up the convergence in the vicinity of the minimum of the cost function.

Other techniques, [9] and [10], which are used in non-linear systems suffer from more or less the same problems. These difficulties indicate the need for an identification system that will operation without supervision, require neither educated guesses nor initialization, be very reliable, and use simple implementation that avoids the troublesome iterative initializing process.

The procedure developed in this thesis makes use of the method of statistical regression analysis, applied previously with success to linear systems, [4], and confined for a long time to the analysis of statistical populations, [11] and [12]. The method of steepest descent is applied to the classical regression analysis in order to speed up the convergence. To avoid the need for reinitialization, the properties of exponentially-mapped=past statistical variables is used, [13].

Although the procedure is applied here to a simplified model of a fast breeder nuclear reactor system, the method is general enough to be applicable to many other automatic control systems that are represented by more complex models.

For the particular nuclear reactor system used as the example application, the variable representing the delayed neutron concentration is a nonmeasurable state and in order to have its value for the identification system a Kalman filter, [14] and [15], was used.

The unknown parameters were identified within 3% of their actual values. The example demonstrates that convergence occurs and that the time of convergence compares favorably with current methods, [16]. The simulation results shown in this thesis were obtained using zero as the initial values for the parameters; however, tests have shown that the convergence time can be greatly improved if an educated guess is accepted.

An executive software routine was written in FORTRAN IV for the XDS-9300 digital computer and was applied both for the control of the process and for establishing the operational performance and accuracy of the Comcor-5000 analog computer that simulated the nuclear reactor system and was used to implement the analog part of the identification procedure.

II. REACTOR DYNAMICS

The point model of the nuclear reactor kinetic equations for U^{235} fuel can be found in several references, [16]-[19].

These equations are

$$\frac{dn}{dt} = \frac{\delta K}{\ell^*} n - \frac{\beta}{\ell^*} n + \sum_{i=1}^{6} \lambda_i C_i \tag{1}$$

and

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell^*} n - \lambda_i C_i \qquad i = 1,\ldots,6. \tag{2}$$

The left-hand term in equation (1) represents the rate of change of the neutron density; the first term on the right side of equation (1) represents the rate of production of prompt neutrons, the second term the rate of absorption plus leakage and the last one, the production rate of delayed neutrons. The variable δK, the reactivity input, can be expressed as

$$\delta K = \rho_e - \sum_{j=1}^{\ell} \alpha_j T_j .$$

Applying this substitution, equations (1) and (2) become

$$\frac{dn}{dt} = \left(\rho_e - \sum_{j=1}^{\ell} \alpha_j T_j\right) \frac{n}{\ell^*} - \frac{\beta}{\ell^*} n + \sum_{i=1}^{6} \lambda_i C_i \tag{3}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell^*} n - \lambda_i C_i \tag{4}$$

where T is defined by

$$\varepsilon_j \frac{dT_j}{dt} = \mu_j \left(\frac{n}{n_0} - 1 \right) - \sum_{k=1}^{\ell} \sigma_{jk} \left(T_j - T_k \right) \quad j=1,\ldots,\ell. \tag{5}$$

Since μ_j and β_i are fractional quantities it follows from their definitions that

$$\sum_{j=1}^{\ell} \mu_j = 1$$

and

$$\sum_{i=1}^{6} \beta_i = \beta.$$

The initial conditions for equations (3) thru (5) are

$$n(0) = n_o$$

$$C_i(0) = C_{io} = \frac{\beta_i \, n_o}{\ell^* \, \lambda_i} \qquad i = 1,\ldots,6$$

$$T_j(0) = 0 \qquad j = 1,\ldots,\ell.$$

For the purpose of this study a single-time-constant feedback reactor with a single group of delayed neutron precursors will be used; the following equations define the system

$$\frac{dn}{dt} = \left(\rho_e - \alpha T \right) \frac{n}{\ell^*} - \frac{\beta}{\ell^*} n + \lambda C \tag{6}$$

$$\frac{dC}{dt} = \frac{\beta}{\ell^*} n - \lambda C \tag{7}$$

$$\varepsilon \frac{dT}{dt} = \mu \left(\frac{n}{n_0} - 1 \right) - \sigma T. \tag{8}$$

The initial conditions are

$$\dot{\rho}_e(0) = 0$$

$$n(0) = n_o$$

$$C(0) = C_o \quad \frac{\beta\ n_o}{\ell^*\lambda}$$

$$T(0) = 0.$$

By changing variables as indicated in [16], (see Appendix A) a more suitable representation is

$$\dot{N} = \frac{\beta}{\ell^*} \left[(K_e - aT - 1)\ N + D \right] \tag{9}$$

$$\dot{D} = \lambda\ (N - D) \tag{10}$$

$$\dot{T} = \gamma\ (N - 1 - bT), \tag{11}$$

where

$$N = \frac{n}{n_o}\ ,\ D = \frac{C}{C_o}\ ,\ K_e = \frac{\rho_e}{\beta}\ ,\ \frac{\beta}{\ell^*} = 6\text{x}10^3\ ,\ a = \frac{\alpha}{\beta}\ ,\ \gamma = \frac{\mu}{\varepsilon}$$

$$b = \frac{\sigma}{\mu}\ ,$$

and the initial conditions are

$N(0) = 1$ $\qquad$ $T(0) = 0$

$D(0) = 1$ $\qquad$ $K_e(0) = 0.$

Experience with present day design of nuclear reactors indicates that these systems are stable under normal operating conditions. The large breeder reactors are still in the design stage and a computer simulation analysis has shown that these plants will be stable, [1]. Nonetheless, a stability analysis applying the method of Liapunov has been performed for this model and is included in Appendix B.

The model defined by equations (9) thru (11) was implemented, but acceptable accuracy could not be obtained. The large value of β/ℓ^* required a large scaling factor and also a large gain for the state variable N. Because of these two reasons the voltage that represented the state variable N become too small and too close to the uncertainty level (fourth digit) of the analog computer.

From simulation it is observed that $\dot{N}$ is approximately equal to zero. This and the fact that β/ℓ^* is very large implies that equation (12) is a satisfactory approximation for equation (9). This further simplification will, of course, introduce some inaccuracies since it is assumed, in fact, that $\dot{N}$ is zero; however, it has been shown, [16], that the largest error introduced with this assumption is less than 2%.

With this simplification the model is

$$N = - D/(K_e - aT - 1) \tag{12}$$

$$\dot{D} = \quad (N - D) \tag{13}$$

$$\dot{T} = T(n - 1) - b\gamma T. \tag{14}$$

In state variable form where

$$x_1 = D \quad , \quad x_2 = T$$

$$\dot{x}_1 = \lambda\ (x_3 - x_1) \tag{15}$$

$$\dot{x}_2 = \gamma\ (x_3 - 1) - b\gamma x_2 \tag{16}$$

and the variable x_3 is defined as

$$x_3 = - x_1\ /(K_e - ax_2 - 1). \tag{17}$$

For a numerical example, the known constants are assumed to be $\lambda = 0.4$ and $\gamma = 1.0$ and the parameters to be identified are "a" and "b". Using these values the equations are

$$\dot{x}_1 = 0.4\,(x_3 - x_1) \tag{18}$$

$$\dot{x}_2 = x_3 - 1 - bx_2 \tag{19}$$

and the variable x_3 is defined in equation (17). The analog computer equations using the scaled variables

$$\dot{\bar{x}}_1 = 40x_1$$
$$\dot{\bar{x}}_2 = 40x_2$$

and

$$\dot{\bar{x}}_3 = 2x_3$$

are then

$$\dot{\bar{x}}_1 = 8x_3 - 0.4\bar{x}_1 \tag{20}$$

$$\dot{\bar{x}}_2 = 20x_3 - 40 - b\bar{x}_2 \tag{21}$$

where the variable x is defined by

$$\bar{x}_3 = -\frac{2\bar{x}_1}{40K_e - a\bar{x}_2 - 40} \tag{22}$$

Figure 2 shows the implementation of the above equations and the settings of the potentionmeters for this scaling are shown in Table I.

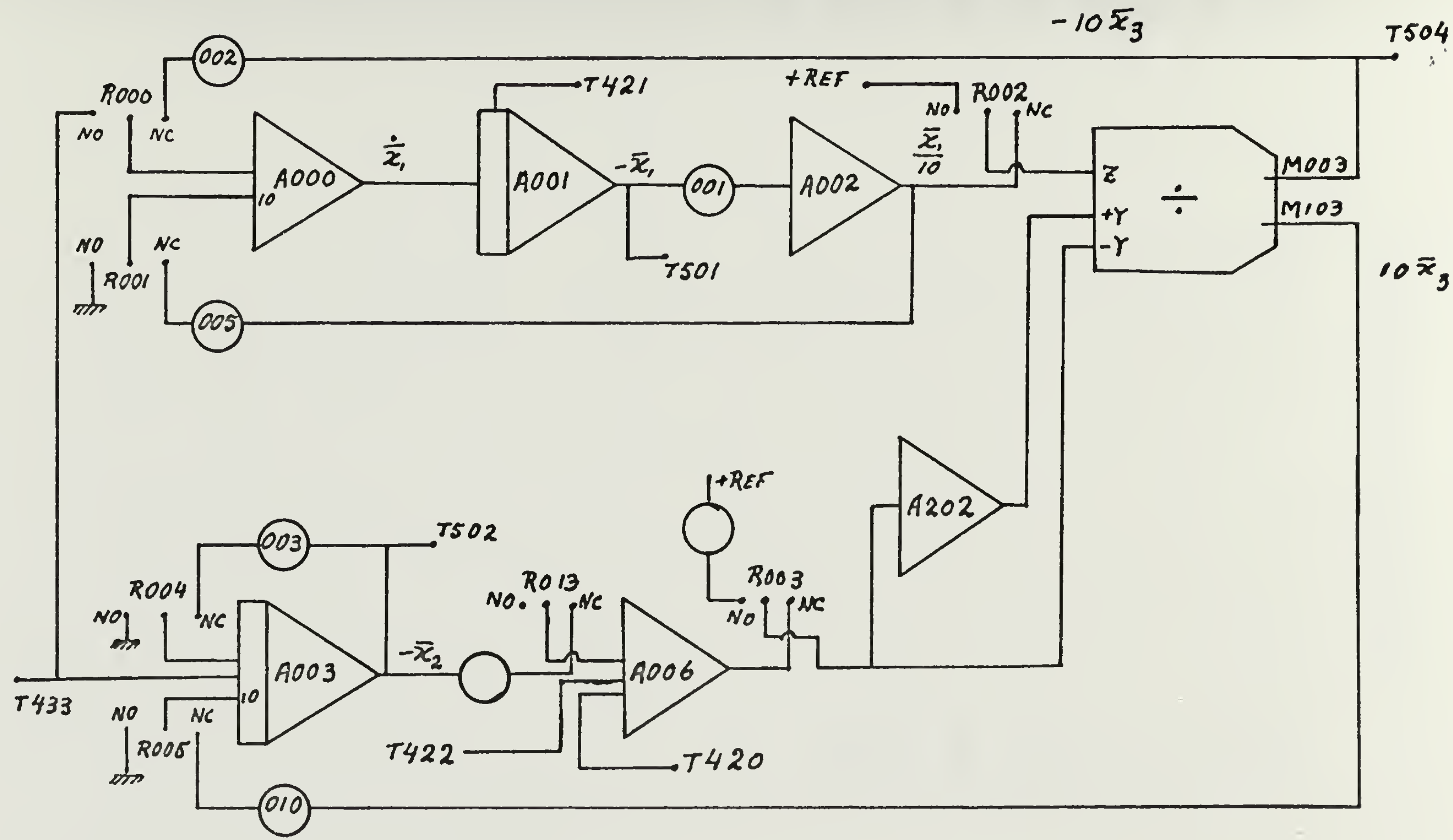

Figure 2. Analog implementation of the nuclear reactor simulation.

TABLE I

POTENTIOMETER SETTINGS

Pot	Value	Pot	Value
P001	0.200	P012	0.200
P002	0.800	P015	0.600
P003	0.500	P017	0.600
P004	0.500	P021	0.600
P005	0.200	P051	0.600
P006	0.400	P055	0.600
P010	0.200	P057	0.600
P011	0.600		

III. REGRESSION ANALYSIS

Several attempts were made in this study to achieve the desired goals for the identification system as stated in Section I. Arrising in all of these attempts was the problem stated by R. C. K. Lee, [14], and illustrated in Figure 3, which is related to the closed-loop formed by the estimator and parameter identification system. This problem is difficult because the overall system is nonlinear even for linear plants. Because convergence of this loop was not proved, and because estimation and identification of parameters for nonlinear cases depends strongly on the initial value[2], it was decided to approach the problem in the following alternative way.

The system is nonlinear as defined by equations (9) thru (11). However, the simplification performed on the model that leads to equations (18) and (19) yields a second-order linear model with the variable x_3 representing an input. Since it is possible to measure N and T, which are represented in the simplified model by the variable x_3 and the state variable x_2, respectively, then the non-measurable state D, represented by x_1, can be easily estimated. In order to make the simulation realistic, the variable x_3 is treated as a non-deterministic forcing function. This function is generated by applying equation (22) and the resulting

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[2] It is possible for this kind of system to reach a steady-state value different from the actual value (this is known as a state of nonlinear stability).

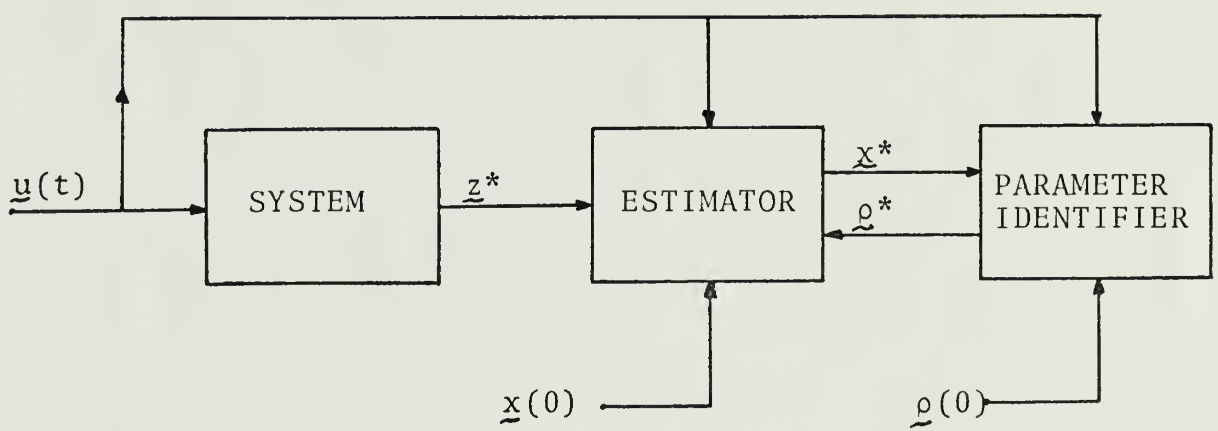

Figure 3. Block diagram of a cascade estimator-identifier.

value is corrupted with some Gaussian noise of mean zero obtained from the low-frequency Gaussian noise generator in the Comcor-5000 analog computer.

This simplification has the advantage that it opens the estimator-identification loop, Figure 4, because the only estimated state (x_1) is neither a function of the unknown parameter "a", nor a function of the unknown parameter "b".

The technique used in this study, known as statistical regression analysis, consists basically of finding the best fit to a set of data using a least-squares criterion. It has been shown, [4] and [10], how the steepest descent method can be used on the analog computer for parameter optimization and model building, and therefore this method was applied in the derivation of the regression equations (27) and (30).

In order to perform the regression analysis it is required to define S, a cost function. The cost function, for simplicity, is usually chosen to be the integral of the square of the error, which is defined by a relationship that contains the parameter to be identified. The second step is to find the rate of change of S with respect to a new independent variable[3], τ, and to minimize the cost function using the method of steepest descent. The details of these methods are developed in the following sections.

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[3] The variable τ represents computer time and it changes more rapidly than real time.

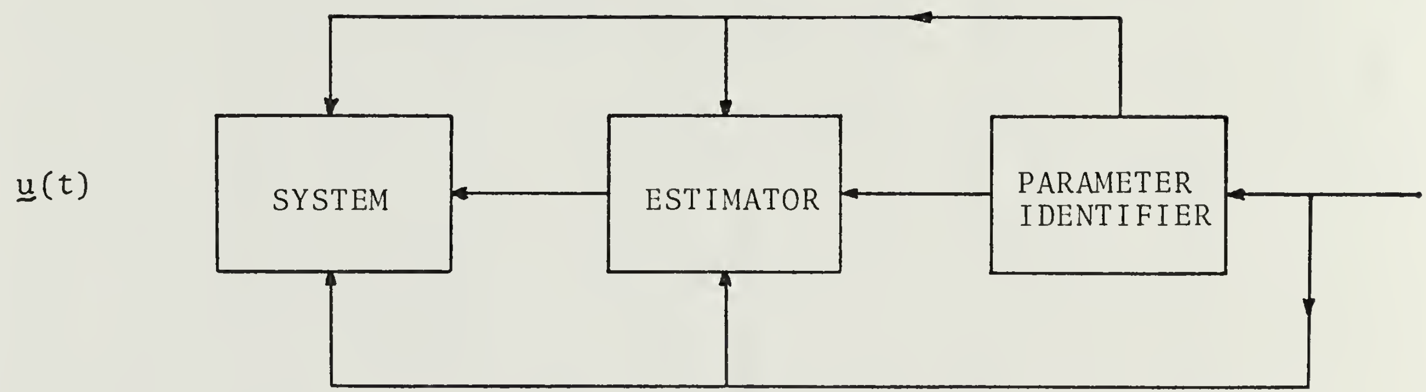


Figure 4. Block diagram of the estimator-identifier.

A. IDENTIFICATION PROCEDURE FOR THE UNKNOWN PARAMETER B

From equation (14) an error function ε_1 can be defined as

$$\varepsilon_1 \triangleq \dot{T} - \gamma(N-1-\hat{b}T) \qquad (23)$$

and a cost function S_1 as

$$S_1 \triangleq \int_0^t \varepsilon_1^2 \, dt = \int_0^t [\dot{T}-T(N-1-\hat{b}T)]^2 \, dt \qquad (24)$$

$\hat{b}$ is an estimate of the parameter b.

The partial derivative of S_1 with respect to time is

$$\frac{\partial S_1}{\partial \tau} = \frac{\partial S_1}{\partial \hat{b}} \times \frac{\partial \hat{b}}{\partial \tau} .$$

Using the method of steepest descent applied directly to the objective function it is required, [4], to make

$$\frac{\partial \hat{b}}{\partial \tau} = \frac{\partial S_1}{\partial \hat{b}} .$$

This ensures that as τ increases the estimate $\hat{b}$ changes in such a way that S_1 decreases. When the estimate b makes $\partial\hat{b}/\partial\tau = 0$ the minimum of the cost function has been reached.

Taking the partial derivative of equation (24) with respect to $\hat{b}$ gives

$$\frac{\partial S_1}{\partial \hat{b}} = 2\int_0^t [\dot{T}- \gamma(N-1-bT)]\gamma \; T \, dt$$

and, therefore

$$\frac{\partial \hat{b}}{\partial \tau} = -2\gamma\left[\int_0^t \dot{T}t dt - \gamma\int_0^t NTdt + \gamma\int_0^t Tdt + b\int_0^t T\, dt\right]. \quad (25)$$

Letting

$$\begin{aligned} x_1 &= D \\ x_2 &= T \\ x_3 &= N \end{aligned} \quad (26)$$

and substituting (26) into equation (25) gives the final expression

$$\frac{\partial b}{\partial \tau} = 2\gamma\left[\int_0^t \dot{x}_2 x_2 dt - \gamma\int_0^t x_2 x_3 dt + \gamma\int_0^t x_2 dt + \hat{b}\int_0^t x_2^2 dt\right]. \quad (27)$$

Since x_2 and x_3 are both measurable and continuously available, the identification of the unknown parameter "b" could also be done continuously and the input to the identification system does not require any digital processing. Due to a lack of analog multipliers, however, all multiplications were performed in the digital computer. Figure 5 shows the implementation diagram for equation (27). In Section III-D a modification to the procedure is introduced in order to avoid the need for reinitialization; this modification uses the properties of exponentially-mapped-past averages.

B. IDENTIFICATION PROCEDURE FOR THE UNKNOWN PARAMETER A

The parameter "a" can be identified using equation (12). Since D is a non-measurable state and it is needed in the

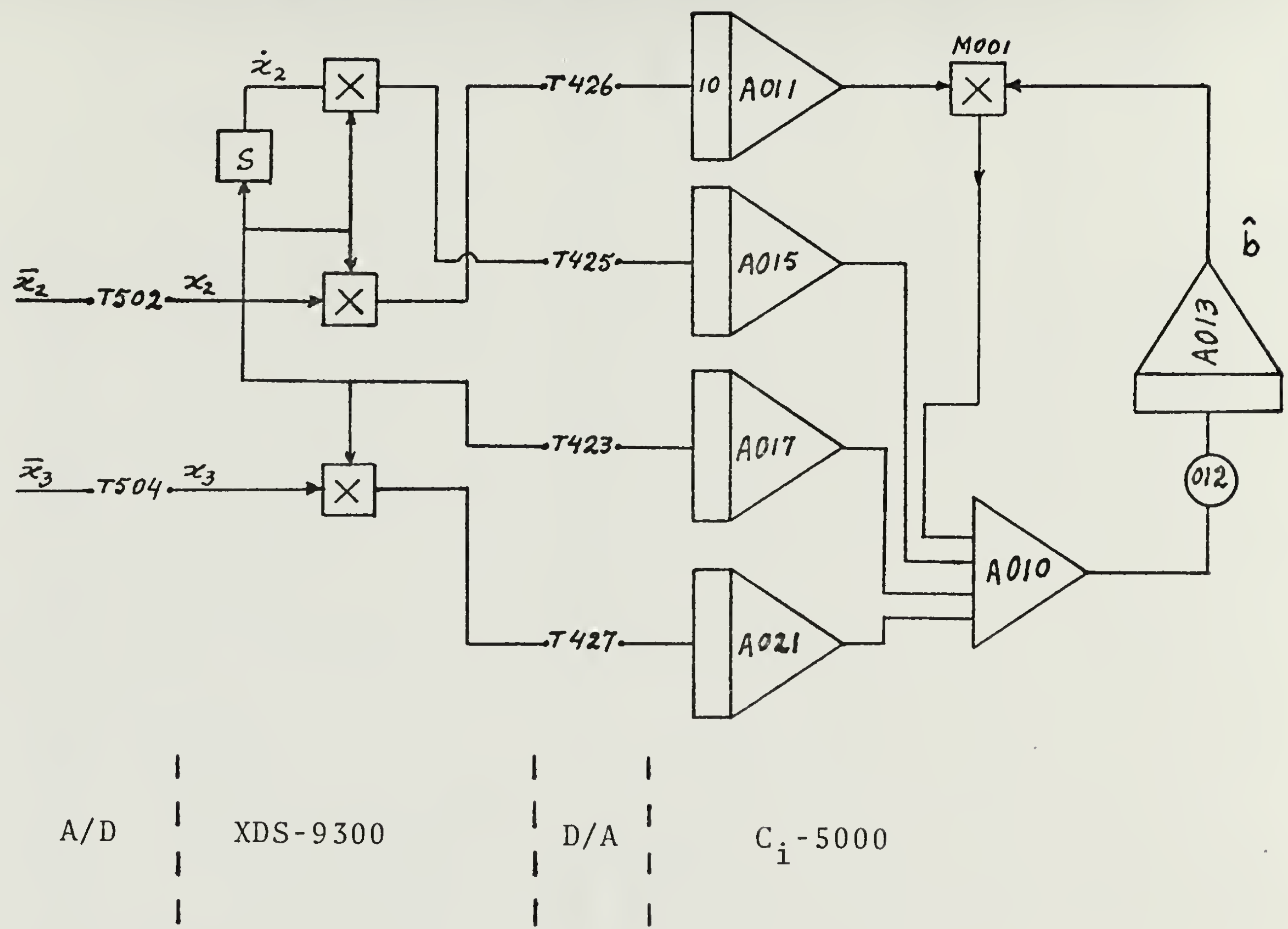


Figure 5. Identification system for parameter "b" using regression analysis and steepest descent only.

regression equation, a Kalman filter, as described in Section III-C, was used to obtain the estimate of the variable D.

Taking equation (12)

$$N = - D/(K_e - aT - 1)$$

and defining an error criterion ε_1 as

$$\varepsilon_2 \triangleq D + N(K_e - \hat{a}T - 1), \tag{28}$$

leads to the cost function

$$S_2 = \int_0^t \varepsilon_2^2 \, dt. \tag{29}$$

a is an estimate of the parameter "a". The steepest descent algorithm can be applied to obtain the differential equation that defines the regression analysis for the unknown parameter "a".

Using (26) gives

$$\varepsilon_2^2 = [x_1 + x_3 \, (K_e - \hat{a} \, x_2 - 1)]^2 ,$$

and

$$\frac{\partial S_2}{\partial \hat{a}} = \frac{\partial}{\partial \hat{a}} \int_0^t [x_1 + x_3 \, (K_e - \hat{a} \, x_2 - 1)]^2 \, dt$$

$$= 2 \int_0^t [x_1 + x_3 \, (K_e - \hat{a} \, x_2 - 1)](-x_2 x_3) \, dt.$$

Therefore,

$$\frac{\partial \hat{a}}{\partial \tau} = 2 \left[\int_0^t x_1 \, x_2 \, x_3 \, dt + \int_0^t K_e \, x_2 \, x_3^2 \, dt \right.$$

$$- \hat{a} \int_0^t x_3^2 \, x_2^2 \, dt - \int_0^t x_3^2 \, x_2 dt \Bigg]. \qquad (30)$$

Figure 6 shows the analog implementation of equation (30).

As in the identification of parameter "b", (Section III-A), this implementation was augmented by using the properties of exponentially-mapped-past variables (see Section III-D). This was done in order to avoid the troublesome iterative reinitialization.

C. KALMAN FILTER

As stated in Section II, the linear equations (15) and (16) are used in the Kalman filter for the estimation of the state variable D. These equations are repeated here for convenience.

$$\dot{x}_1 = \lambda(x_3 - x_1) \qquad (15)$$

$$\dot{x}_2 = \gamma(x_3 - 1 - bx_2). \qquad (16)$$

In the application of the Kalman filter the variable x_3 is treated as a forcing function.

Figure 7 shows the block diagram of the estimator for the system defined by the difference equations

$$\underset{\sim}{x}(k=1) = \underset{\sim}{\Phi} \, \underset{\sim}{x} \, (k) + \underset{\sim}{\Gamma} \, \underset{\sim}{w} \, (k)$$

$$\underset{\sim}{z}(k) = \underset{\sim}{H} \, \underset{\sim}{x} \, (k) + \underset{\sim}{v} \, (k)$$

where k represents a point in the discrete time domain, $\underset{\sim}{x}(k)$ is the vector of the state variables at time k, $\underset{\sim}{w}(k)$ is the system input which in this case is the sum of two components,

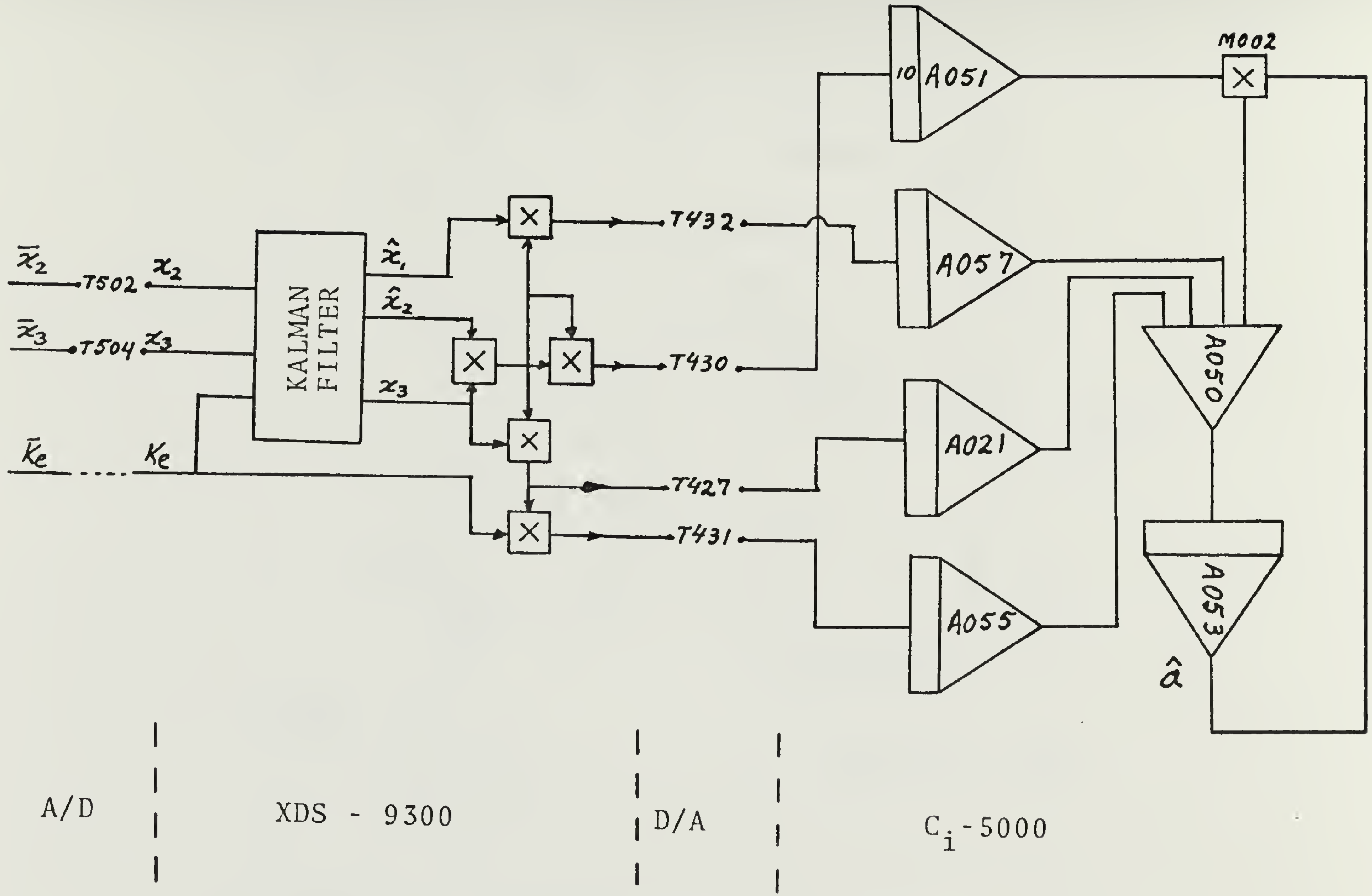


Figure 6. Identification system for parameter "a" using regression analysis and steepest descent only.

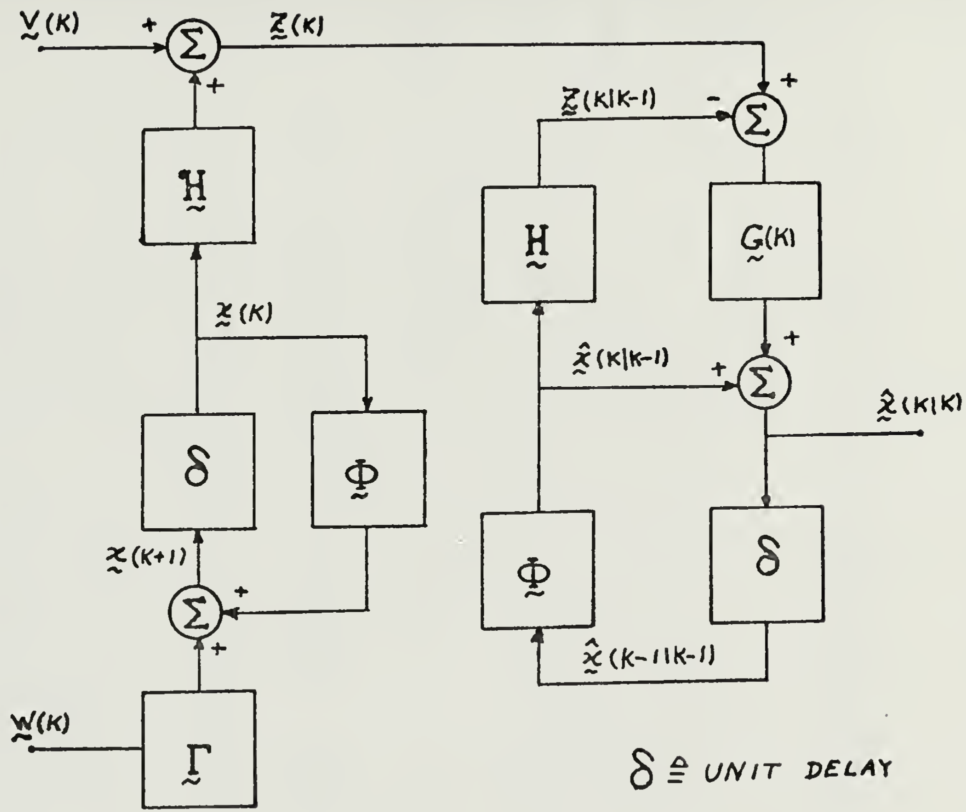


Figure 7. Block diagram of the Kalman filter.

the variable x_3 (a deterministic input) and Gaussian noise. $\underset{\sim}{z}(k)$ is the vector of measured states at time k, and $\underset{\sim}{v}(k)$ is the measurement noise. The matrices $\underset{\sim}{\Phi}$, $\underset{\sim}{\Gamma}$ and $\underset{\sim}{H}$ are, respectively, the state transition matrix, the distribution matrix and the measurement matrix.

Recurrence relations for the optimal filter, [14], [15], are

$$\underset{\sim}{x}(k|k) = \hat{\underset{\sim}{x}}(k|k-1)+\underset{\sim}{G}(k)[\underset{\sim}{z}(k)-\underset{\sim}{H}\ \hat{\underset{\sim}{x}}(k|k-1)]$$

$$\hat{\underset{\sim}{x}}(k+1|k) = \underset{\sim}{\Phi}\ \underset{\sim}{x}(k|k),$$

where $\hat{\underset{\sim}{x}}(k|k)$ is the estimated state vector at time k using observations up to and including time k, $\hat{\underset{\sim}{x}}(k|k-1)$ is the predicted state vector at time k, using observations up to time k-1 and $\underset{\sim}{G}(k)$ is the gain matrix.

The gain matrix $\underset{\sim}{G}(k)$ is given by

$$\underset{\sim}{G}(k) = \underset{\sim}{P}(k|k-1)\underset{\sim}{H}^T[\underset{\sim}{H}\ \underset{\sim}{P}(k|k-1)\underset{\sim}{H}^T + R]^{-1}$$

P(k|k-1) is the covariance matrix of prediction error defined by

$$\underset{\sim}{P}(k|k-1) \triangleq E\{[\underset{\sim}{x}(k) - \hat{\underset{\sim}{x}}(k|k-1)][\underset{\sim}{x}(k) - \hat{\underset{\sim}{x}}(k|k-1)]^T\}.$$

The auxiliary equations that determine the propagation of $\underset{\sim}{P}$ are

$$\underset{\sim}{P}(k|k) = (\underset{\sim}{I} - \underset{\sim}{G}(k)\ \underset{\sim}{H})\ \underset{\sim}{P}(k|k-1)$$

$$\underset{\sim}{P}(k+1|k) = \underset{\sim}{\Phi}\ \underset{\sim}{P}(k|k)\ \underset{\sim}{\Phi}^T + \underset{\sim}{Q}$$

where $\underset{\sim}{P}(k|k)$, called the covariance matrix of estimation error, is defined by

$$\underset{\sim}{P}(k|k) \triangleq E\{[\underset{\sim}{x}(k) - \hat{\underset{\sim}{x}}(k|k)][\underset{\sim}{x}(k) - \hat{\underset{\sim}{x}}(k|k)]^T\}.$$

R, the covariance matrix of the measurement noise, and Q, the covariance matrix of the system input noise, are defined as

$$\underset{\sim}{R} = E[\underset{\sim}{v}(k)\ \underset{\sim}{v}^T(k)]$$

$$\underset{\sim}{Q} = \underset{\sim}{\Gamma}\, E[\underset{\sim}{w}(k)\ \underset{\sim}{w}^T(k)]\underset{\sim}{\Gamma}^T.$$

The matrix form of the difference equations, that correspond to the differential equations (15) and (16), used for the filter, is

$$\underset{\sim}{x}(k+1) = \begin{bmatrix} .992 & 0 \\ 0 & .990 \end{bmatrix} \underset{\sim}{x}(k) + \begin{bmatrix} .008 & 0 \\ 0 & .02 \end{bmatrix} \begin{bmatrix} x_3 \\ x_3 - 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

for a sampling interval of .02 sec. and a value of .5 for the parameter "b".

Since "b" is required for the Φ matrix then the identification of "a" is done after the identification of $\hat{b}$ has been completed.

The signal-to-noise ratio is specified as the ratio of two exponentially-mapped-past averages, the square of the signal and the square of the noise signal. This average function is explained in the next section.

The computer listing of the estimator algorithm containing the complete set of equations that characterize the estimator is within the subroutine ESTIM.

D. THE EXPONENTIALLY-MAPPED-PAST AVERAGE FUNCTION

The open-loop approach used for the implementation of the Kalman estimator produced system equations which do not depend explicitly on the unknown parameter "a". This implementation of the Kalman estimator allows use of the regression analysis technique. However, this procedure allows the variable x_3 to reach the identifier without being filtered. Another problem still present is the iterative reinitialization.

To solve these problems an exponentially-mapped-past (EMP) average function, [13], was found very useful. The EMP function also was found to be very suitable for acting both as a filter and eliminating the overloading of the integrators used in the identification. This exponential weighting function decreases with early values of time and its implementation allows for relatively fast control of its time constant. It gives the advantages of easy computation and simple implementation.

The EMP average function is defined as

$$f_a(t) = \alpha \int_{-\infty}^{\infty} f(\tau)\ e^{\alpha(\tau - t)} u(t-\tau) d\tau$$

where u(t) is a unit step function. This convolution integral of f(t) and an impulse response of a low-pass RC

filter, with leakage time constant RC = $1/\alpha$ is shown implemented in Figure 8.

Figures 9 and 10 illustrate the changes made in order to include the EMP function into the simulation. The changes are the addition of input potentiometers with settings of α.

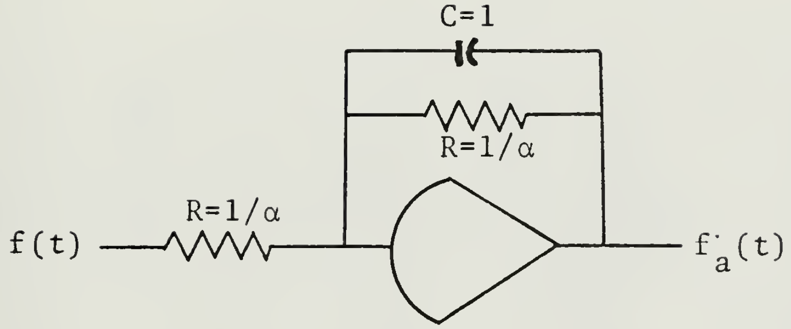


Figure 8. Exponentially-mapped-past average circuit.

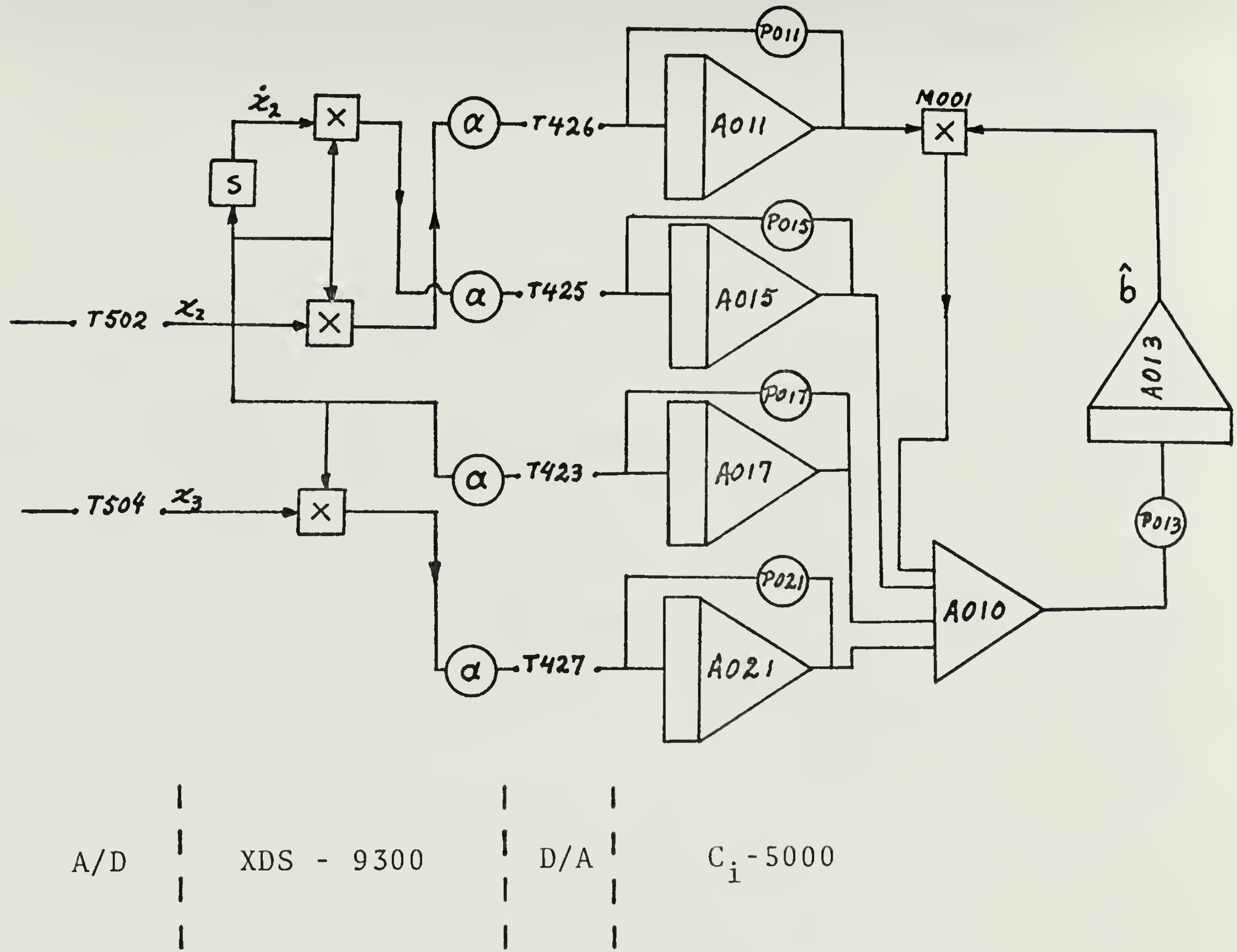


Figure 9. Identification system for parameter "b".

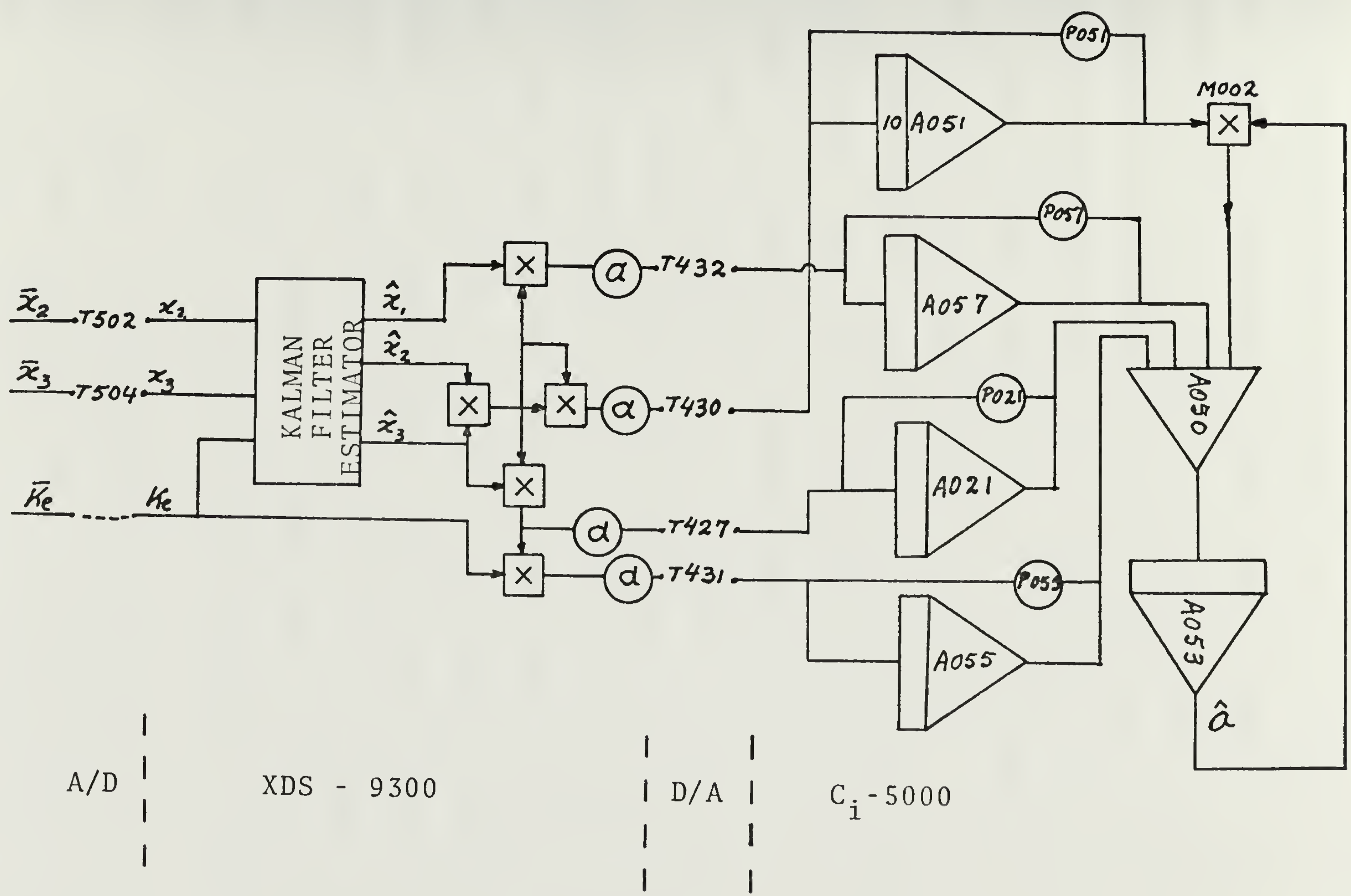


Figure 10. Identification system for parameter "a".

IV. EXECUTIVE ROUTINE

One of the most important tasks prior to obtaining data from the analog simulation is the verification of the operational performance of the analog computer. This is especially critical where the analog computer used in the simulation is large and has many other users, [20]. This task is very difficult and no attempt was made here to cover the whole system; however, it was found worthwhile to check out all components used in this simulation.

The executive routine MAIN can be divided into two subsections: the first one performs the checkout and the second one controls the actual identification.

The test performed by the checkout routines are:

1. TESTO. - This subprogram tests the library subroutines SETPOT, SCAN and DAC. To accomplish this TESTO sets potentiometers and compares the desired setting with the actual setting, allowing for the inherent uncertainty present in the fourth digit. In the same way the digital-to-analog converters (DAC's) are tested for proper setting and also for any interaction among them. Any detected error will cause a message to be printed and a PAUSE to be executed.

2. TEST1. - This subporgram tests the initial condition setting on the plant and the analog-digital converters and digital-analog converters. It takes 1000 samples of each set of converted values and prints the mean errors and the error variances of both tests.

3. TEST2. - This subroutine tests the integrators used in the simulated nuclear reactor system. It prints out the percentage error of the integrator integration rates.

4. TEST3. - This subprogram tests the initial conditions on the simulated plant, which allows verification of the divider M003. It also runs the plant to compare the final value with a precomputed numerical solution. This subroutine prints the percentage error for both tests.

5. TEST4. - This subroutine performs a test of both identification systems using a precomputed numerical solution for the steady-state and prints the percentage errors of both systems compared with the parameter value used in the numerical solution.

The identification of the unknown parameters is performed by subroutines RUN1 and RUN2.

1. RUN1. - Activates the identification system for the parameter "b", which is performed first because the Kalman filter requires the value of the unknown parameter "b". The internal subroutine SAMP1, activated by interrupt 53, performs the sampling of x_2 and x_3, computes the derivative of x_2 by a difference approximation, sets the values of the reactivity input and the required products in the simulated nuclear plant and identification system respectively. RUN1 prints on the teletypewriter the value of "b".

2. RUN2. - Activates the identification system for the parameter "a". HANDY, an internal subroutine activated

by interrupt 53, samples x_2 and x_3, calls ESTIM (Kalman filter) to obtain an estimate of x_1, performs the required multiplications and sets these values in the analog computer for the identification system.

The program was written primarily in FORTRAN IV for the XDS-9300; some assembler language statements were used in order to be able to use the library subroutine DELAY.

A sample of the output taken under conditions of failute of the library hybrid executive routine is shown as a part of the Computer Output.

V. SIMULATION RESULTS

Several tests were performed for each identification system. The first test of each system is shown in Figure 11 and Figure 12. These figures show the response of the identification system to a constant input whose value was numerically precomputed and corresponds to a steady-state value of the plant, i.e., the digital multipliers shown in Figures 9 and 10 were furnished with these steady-state values. Since these runs were performed without state variable trajectories they do not reflect the previously specified desired objective of an identification system. Nonetheless, for this example the identification system converges to the actual parameter value if supplied with a noiseless discrete sample representing the steady-state value of the plant variables. The time scales are unlabeled since the convergence time can be altered arbitrarily in any noiseless situation by adjusting the system gain, without changing the final result. Figure 15 shows the type of plant transient condition used, a truncated ramp. For the identification of the unknown parameter "b" a low gain was used and for the parameter "a" a high gain was used. These choices of low and high gain were made arbitrarily because for noiseless runs the procedure was not sensitive to gain and either high or low gains produced the same satisfactory results. Here again, for the reason stated previously, there is no need for labeling the time scales.

Figures 16 thru 23 show the behavior of the system under simulated real conditions.

The system response in the identification of the parameter "b" with two different levels of measurement noise is shown in Figure 16, where the noise was added to the variable that represents the plant temperature, x_2.

Figures 17 and 18 show the system response in the identification of parameter "a" with two different levels of measurement noise. Unbiased noise was added to the variable that represents the plant temperature, x_2, and to the variable that represents the neutron density, x_3.

Figures 19 thru 21 were obtained with biased measurement noise, and Figures 22 and 23 show typical inputs to the identification system.

Talbe II summarizes the numerical results for various runs with different values of the parameters "a" and "b".

TABLE II

NUMERICAL RESULTS

PARAMETER		IDENTIFICATION	
a	b	$\hat{a}$	$\hat{b}$
0.4	0.3	0.40	0.29
0.4	0.4	0.40	0.39
0.4	0.5	0.40	0.50
0.4	0.6	0.41	0.61
0.4	0.7	0.40	0.71
0.4	0.8	0.39	0.81
0.2	0.5	0.21	0.50
0.3	0.5	0.29	0.50
0.5	0.5	0.50	0.49
0.6	0.5	0.59	0.50
0.7	0.5	0.69	0.50

The signal-to-noise ratios used in all these runs was 20:1.

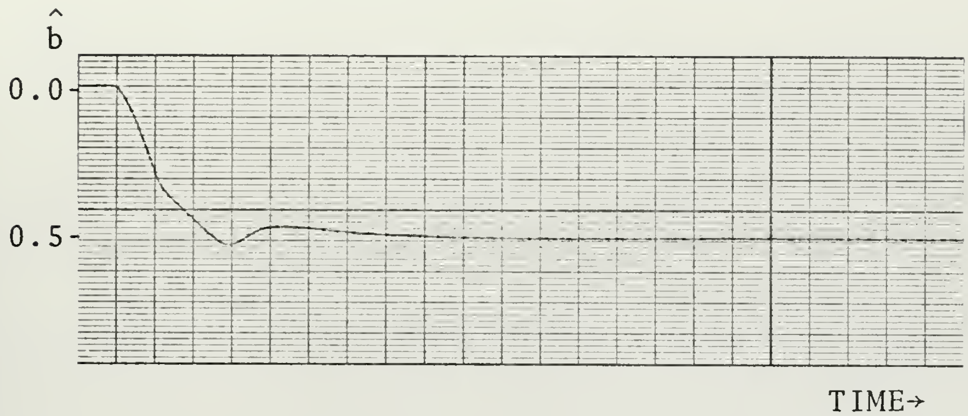


Figure 11. Identification of parameter "b" under steady-state conditions.

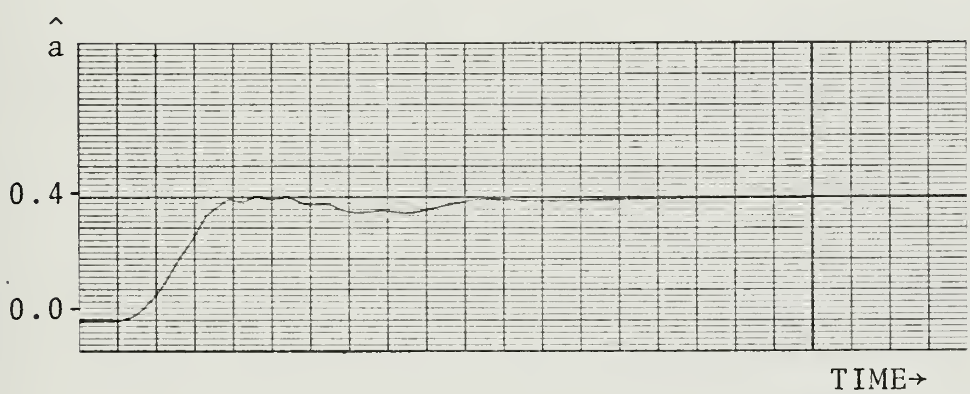


Figure 12. Identification of parameter "a" under steady-state conditions.

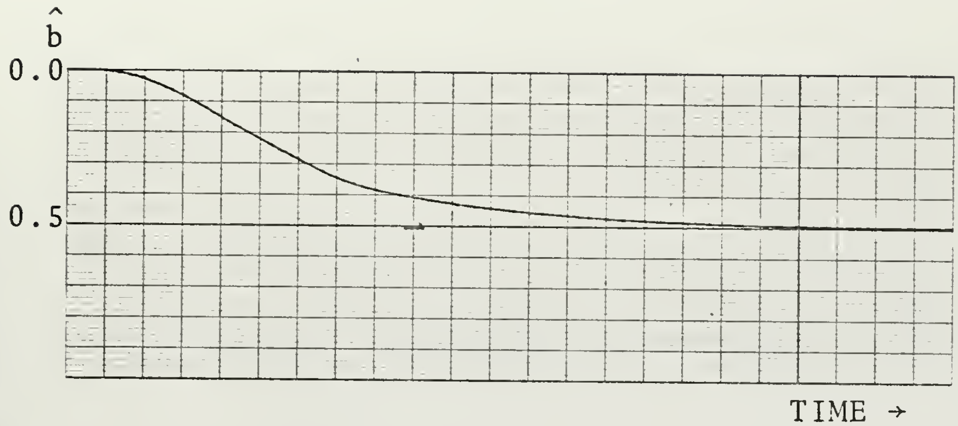


Figure 13. Identification of parameter "b" under transient conditions.

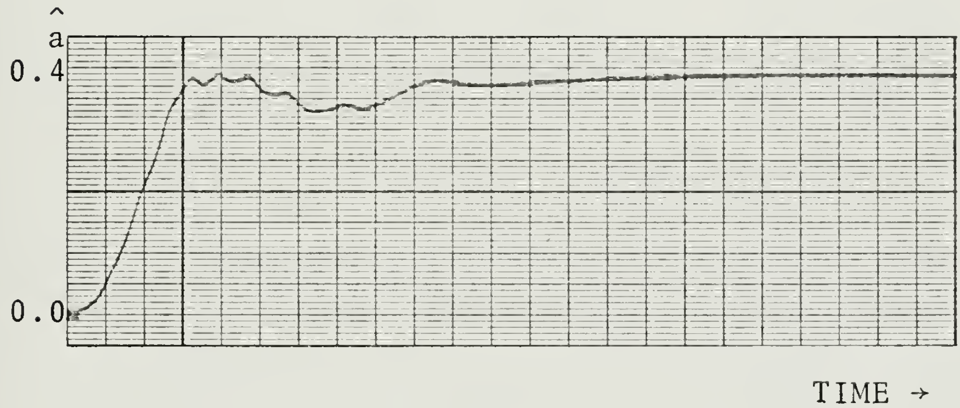
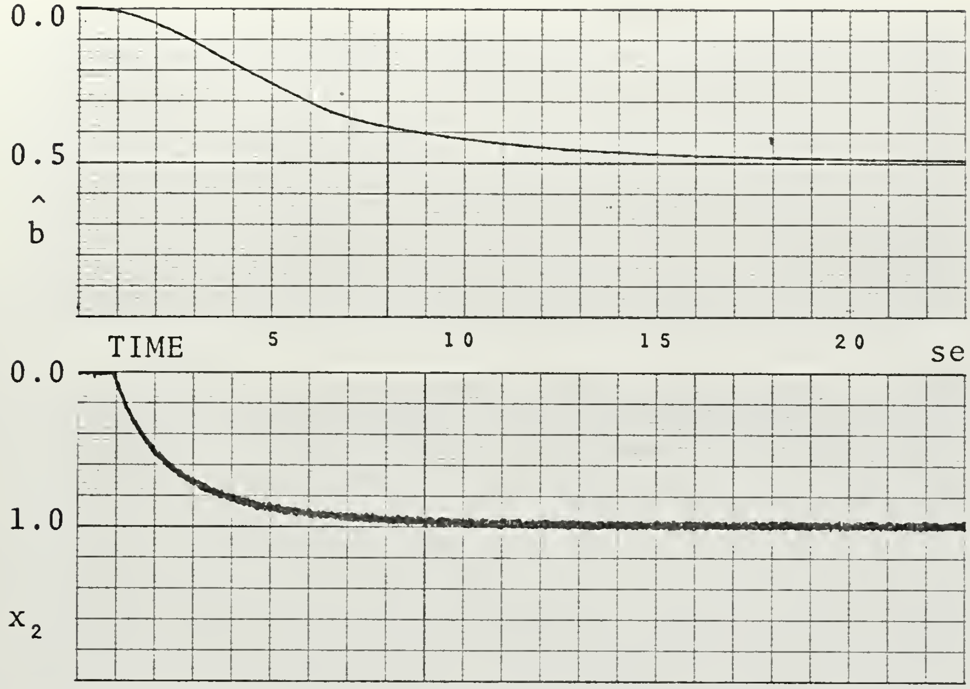
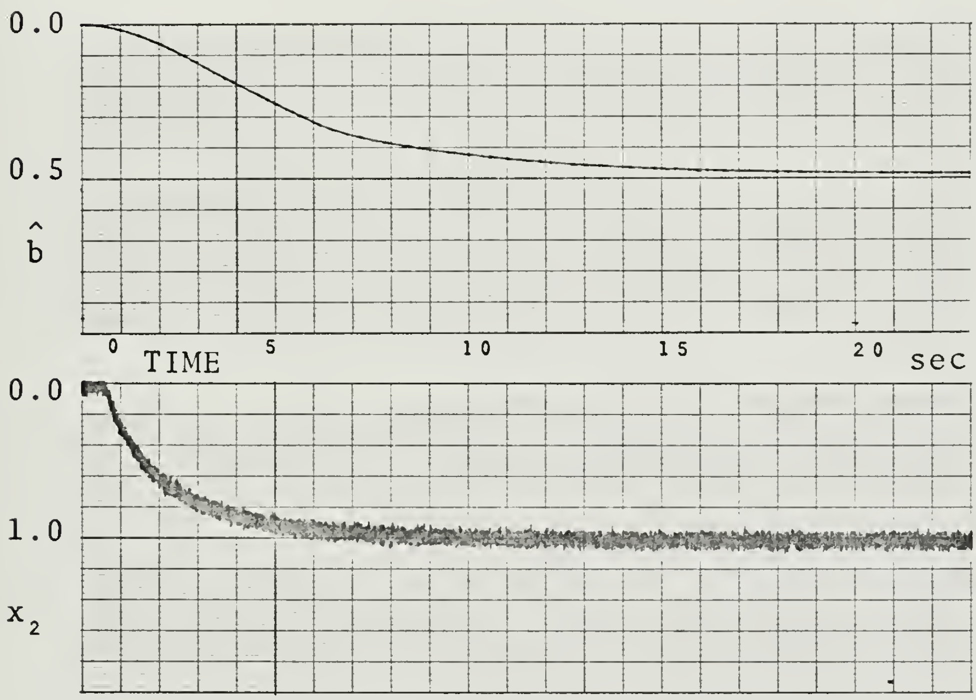


Figure 14. Identification of parameter "a" under transient conditions.

Figure 15. Reactivity function used in the simulation.

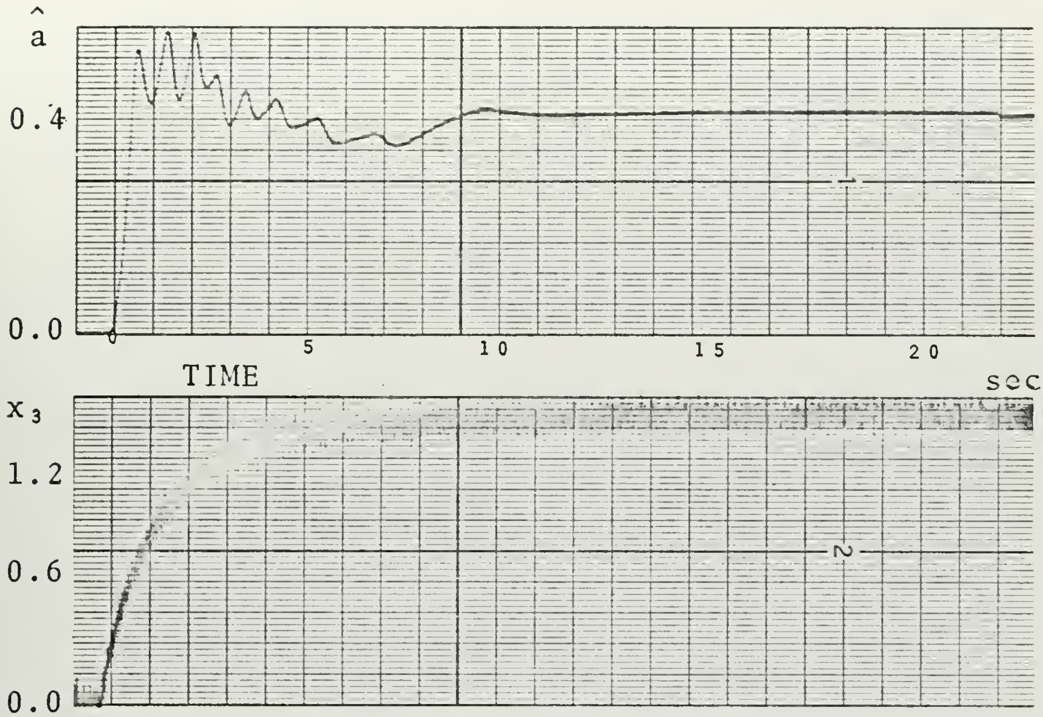


a) Signal to Noise Ratio 20:1

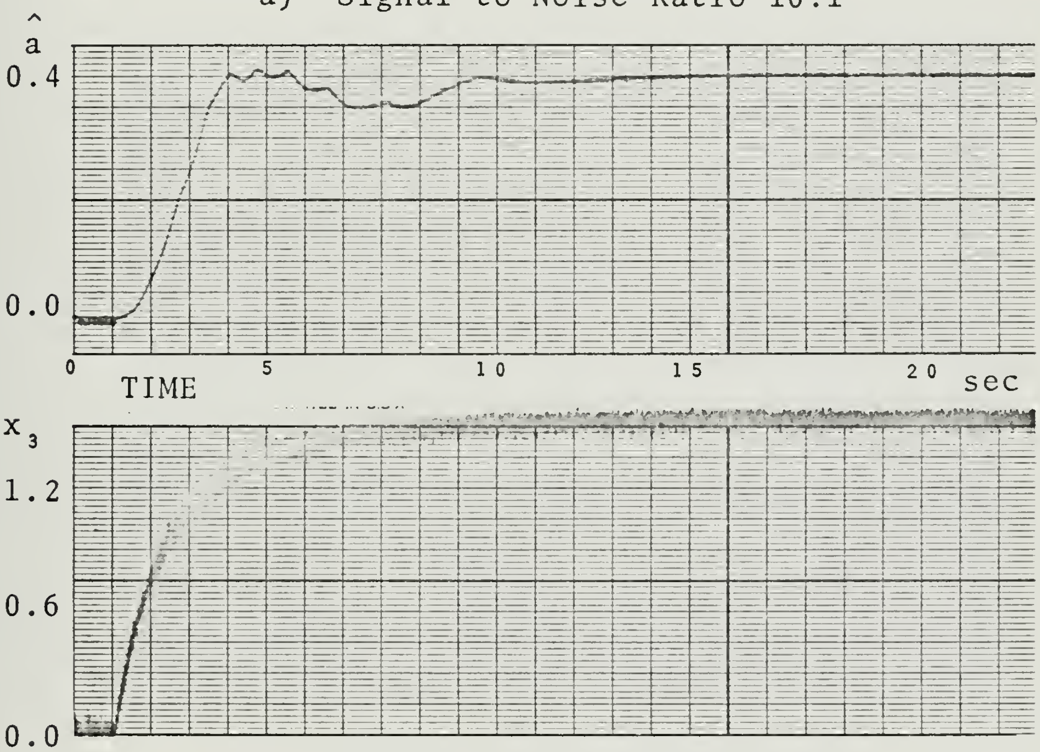


b) Signal to Noise Ratio 10:1

Figure 16. Identification of parameter "b" under noisy conditions.

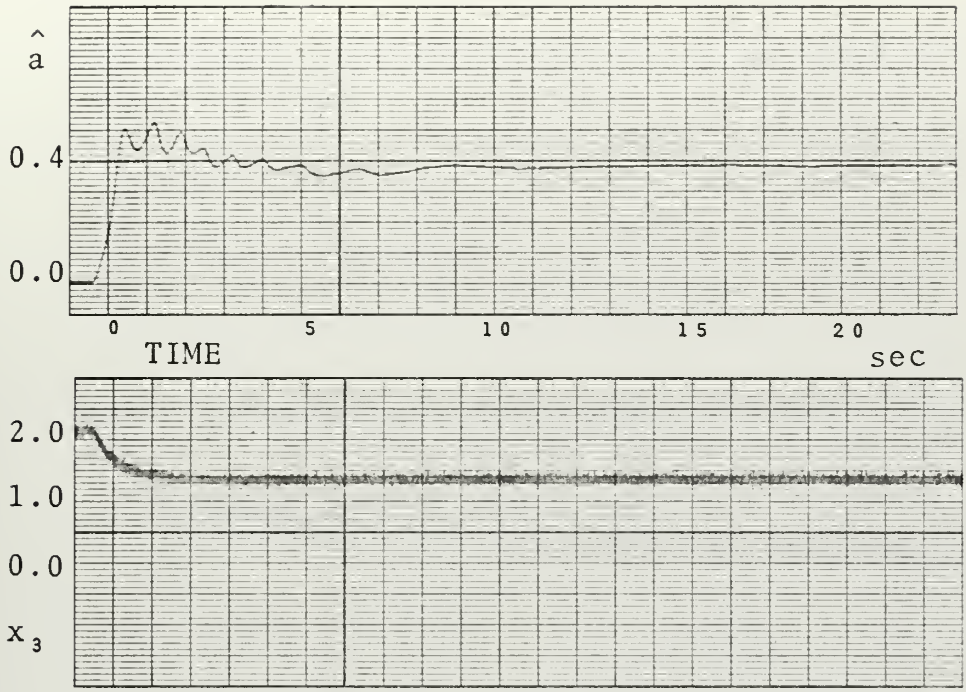


a) Signal to Noise Ratio 10:1

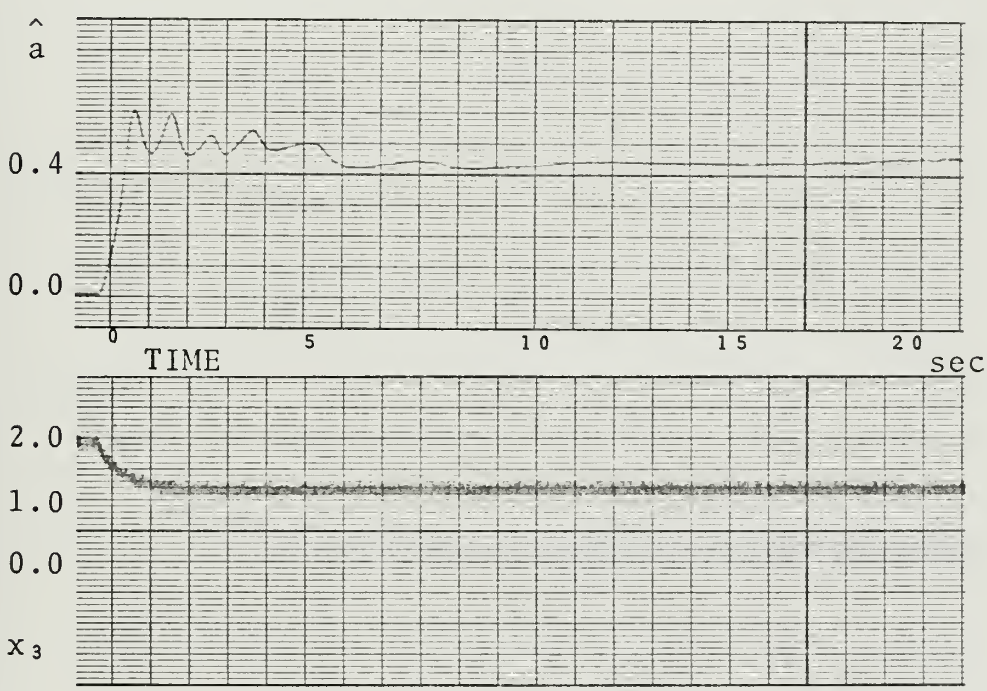


b) Signal to Noise Ratio 15:1

Figure 17. Identification of parameter "a" with measurement noise on x_3.



a) Signal to Noise Ratio 15:1



b) Signal to Noise Ratio 10:1

Figure 18. Identification of parameter "a" with measurement noise on x_2.

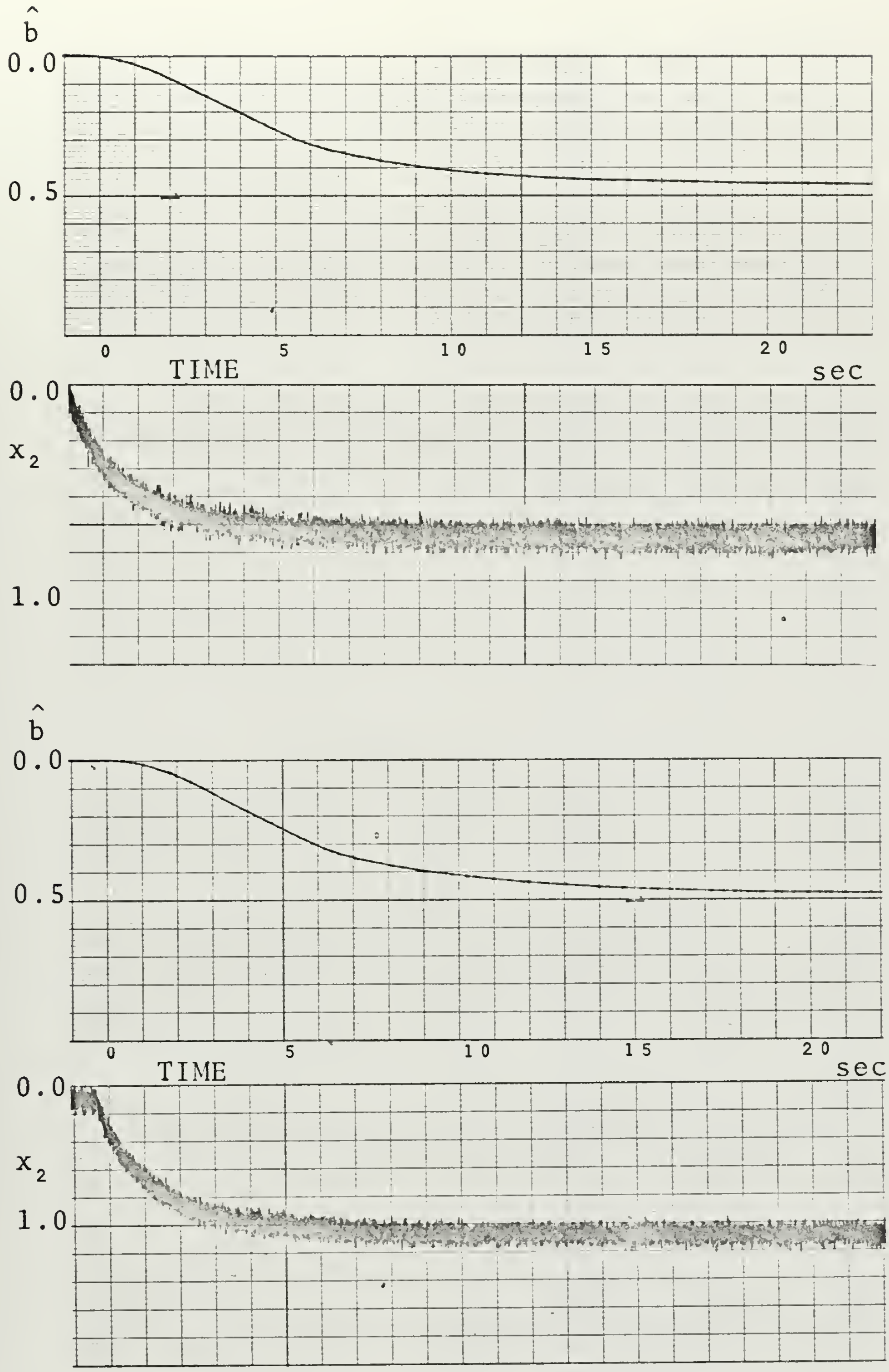


Figure 19. Identification of parameter "b" under a biased noise condition.

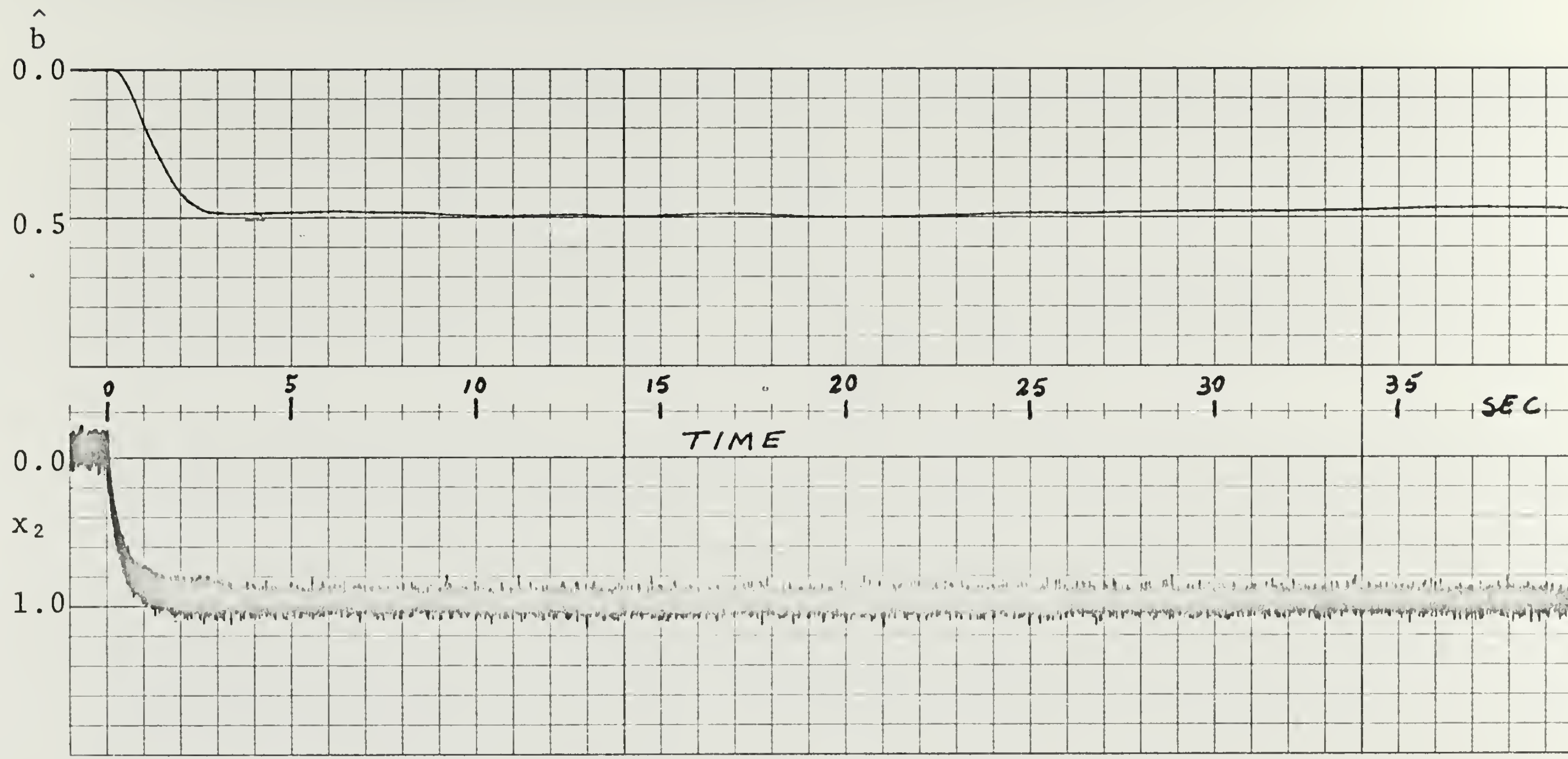


Figure 20. Extreme biased condition for parameter "b".

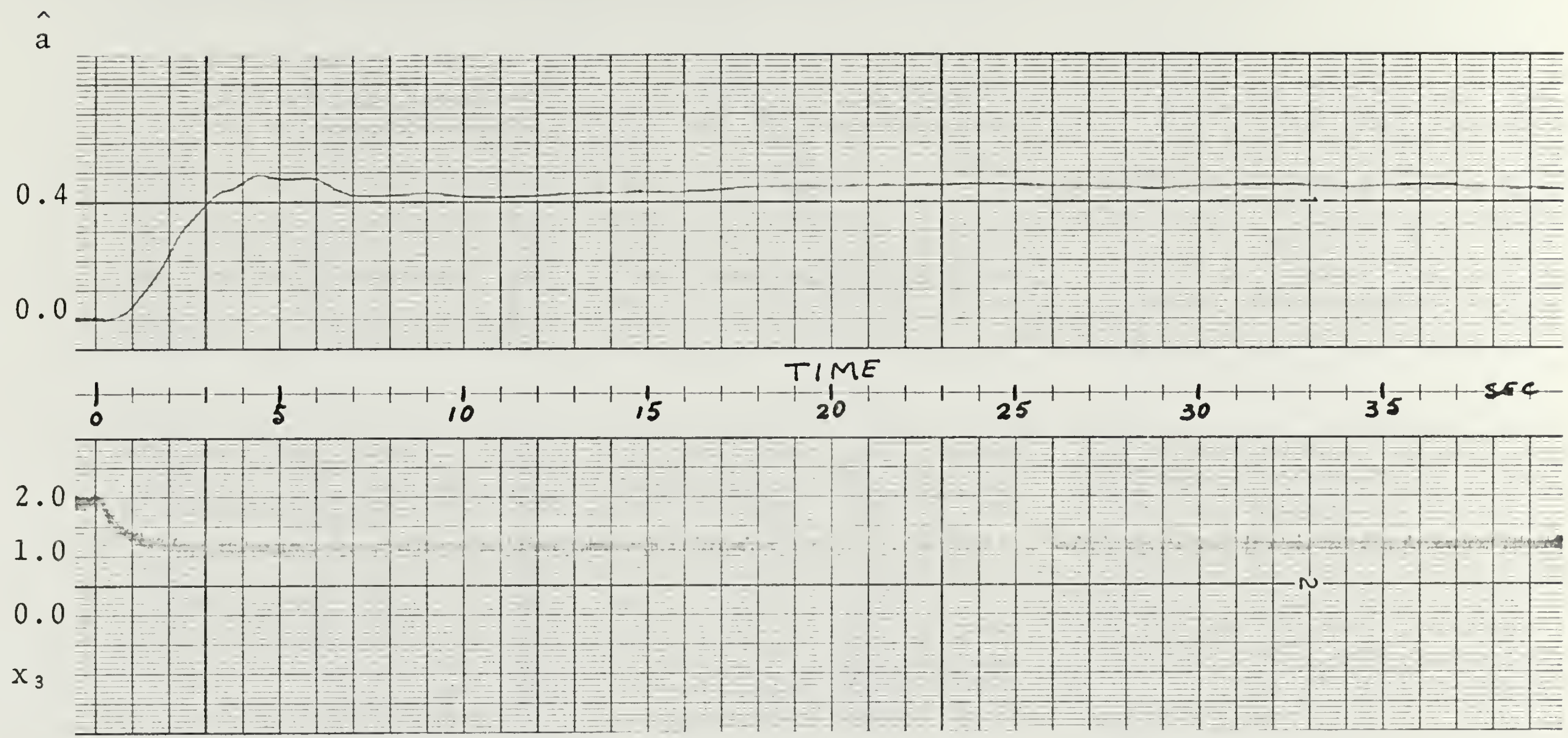


Figure 21. Extreme biased condition for parameter "a".

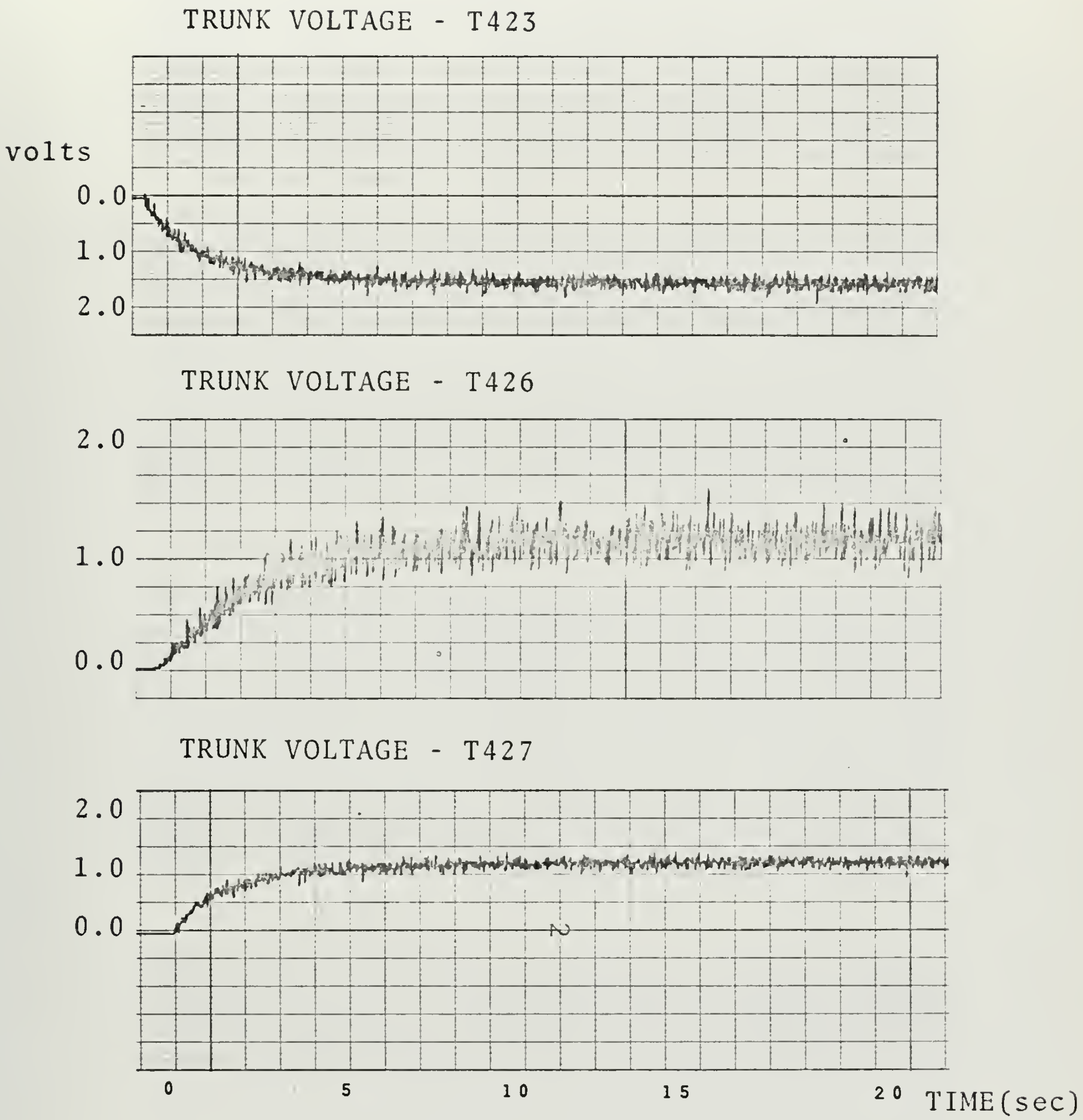


Figure 22. Typical input signals for the identifier of parameter "b".

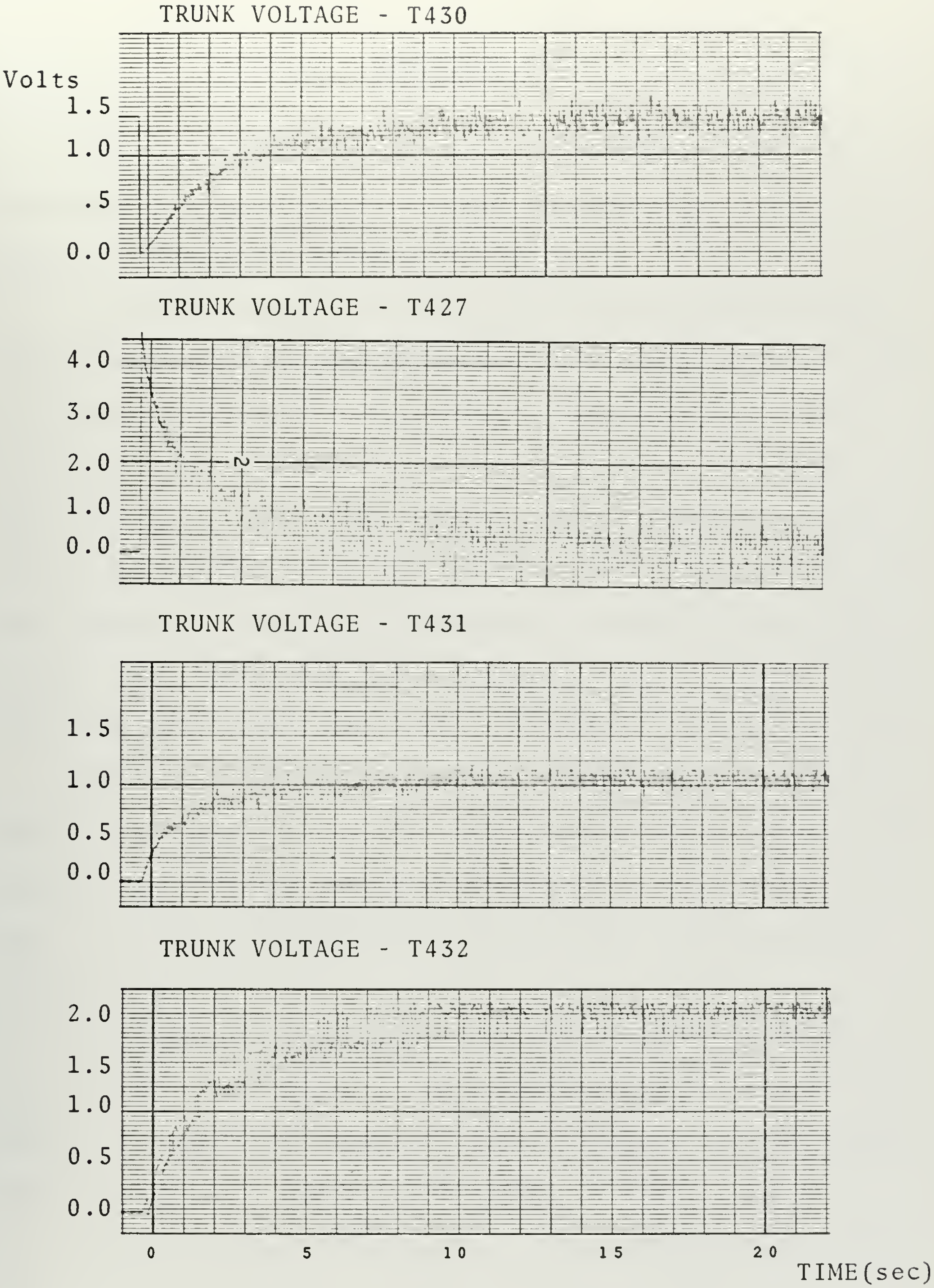


Figure 23. Typical input signals for the identifier of parameter "a".

VI. CONCLUSIONS

From the results of the numerical example used, the following conclusions are reached regarding the desired goals for the identification procedure.

A typical response of the identification system where the initial value of the parameters are set to zero is shown in Figures 16 thru 21. The results indicate that the procedure does not require a guessed value to be effective. Other runs were made with various initial parameter values and if the guessed value was close to the actual value, the convergence time was greatly reduced as might be expected. In any case, whether or not a non zero initial value was used, the system did not require iterative reinitialization.

The simplicity of the identification procedure implementation, together with the small number of required analog components, provides for a very reliable system, since analog integrators and analog multipliers are very reliable components.

The average convergence time for a signal-to-noise ratio of 10:1 was found to be approximately 16 seconds, and about 10 seconds for noiseless measurements. If the signal-to-noise ratio is small the convergence time is large.

For the example considered in this study the procedure identified the unknown parameters within 3% of their actual values with signal-to-noise ratios as low as 10:1 in the measured states.

The identification system for parameter "b" was very insensitive to the noise, except for biased noise (with a mean of 10% of the stead-state value of the state variable), as shown in Figure 20. The extreme biased condition was taken as the maximum noise for which, even when the system does not actually converge, the identifier output oscillates within 4% of the actual parameter value.

The identification of parameter "a" also showed very good performance but it was more affected by measurement noise on x_3 than on x_2. This was expected since x_3 is not filtered by the Kalman filter and x_3 is an important factor in every term of equation (30). This problem can be eliminated by using filtering in the measuring circuits for x_3.

APPENDIX A: DYNAMIC EQUATIONS SIMPLIFICATION

As seen in the case of an heterogeneous reactor made up of n independent media, the state variables are related by the following non linear differential equations:

$$\frac{dn}{dt} = \left(\rho_e - \sum_{j=1}^{\ell} \alpha_j T_j\right) \frac{n}{\ell^*} - \frac{\beta}{\ell^*} n + \sum_{i=1}^{6} \lambda_i C_i \tag{1.1}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell^*} n - \lambda_i C_i \tag{1.2}$$

$$\varepsilon_j \frac{dT_j}{dt} = \mu\left(\frac{n}{n_o} - 1\right) - \sum_{k=1}^{\ell} \sigma_{jk}(T_j - T_k). \tag{1.3}$$

The initial conditions for this set of equations are

$$n(0) = n_o$$

$$C_i(0) = C_{io} = \frac{\beta_i}{\ell^* \lambda_i} n_o$$

$$T_j(0) = 0.$$

For the purpose of this study the complexity of the model is reduced to a single time constant approximation, i.e., i = 1 and j = 1, so that

$$\frac{dn}{dt} = \left(\rho_e - \alpha T\right) \frac{n}{\ell^*} - \frac{\beta}{\ell^*} n + \lambda C \tag{2.1}$$

$$\frac{dc}{dt} = \frac{\beta}{\ell^*} n - \lambda C \tag{2.2}$$

$$\varepsilon \frac{dt}{dt} = \mu\left(\frac{n}{n_o} - 1\right) - \sigma T \tag{2.3}$$

with the following initial conditions

$$\rho_e(0) = 0 \qquad C(0) = C_o = \frac{\beta n_o}{\ell^*\lambda}$$
$$n(0) = n_o \qquad T(0) = 0 .$$

A further manipulation will produce a more convenient form, i.e.,

$$\frac{dn}{dt} = \frac{\beta}{\ell^*} \left[\left(\frac{\rho_e}{\beta} - \frac{\alpha}{\beta} T - 1\right) n + \frac{\lambda\ell^*}{\beta} C\right]$$
$$= \frac{\beta}{\ell^*} \left[\left(\frac{\rho_e}{\beta} - \frac{\alpha}{\beta} T - 1\right) n + \frac{\lambda\ell^*}{n_o} n_o C\right],$$

or

$$\frac{1}{n_o} \frac{dn}{dt} = \frac{\beta}{\ell^*} \left[\left(\frac{\rho_e}{\beta} - \frac{\alpha}{\beta} T - 1\right) \frac{n}{n_o} + \frac{C}{C_o}\right] . \qquad (3.1)$$

In the same way

$$\frac{dc}{dt} = \frac{\beta}{\ell^*} n - \lambda C$$
$$= \lambda \left(\frac{\beta}{\ell^*\lambda} n - C\right)$$
$$= \lambda \left(C_o \frac{n}{n_o} - C\right)$$

and

$$\frac{1}{C_o} \frac{dc}{dt} = \lambda\left(\frac{n}{n_o} - \frac{e}{C_o}\right)$$
$$\qquad (3.2)$$
$$\varepsilon\frac{dt}{dt} = \mu\left(\frac{n}{n_o} - 1\right) - \sigma T$$

$$\frac{dt}{dt} = \frac{\mu}{\varepsilon}\left[\left(\frac{n}{n_o} - 1\right) - \frac{\sigma}{\mu} T\right] . \qquad (3.3)$$

Now if

$$N \triangleq \frac{n}{n_o} \qquad D \triangleq \frac{C}{C_o} \qquad K_e \triangleq \frac{\rho_e}{\beta}$$

$$\gamma \triangleq \frac{\mu}{\varepsilon} = 1 \qquad a \triangleq \frac{\alpha}{\beta} \qquad b \triangleq \frac{\sigma}{\mu} ,$$

then

$$\dot{N} = \frac{\beta}{\ell^*} \left[(K_e - aT - 1) N + D \right] \tag{4.1}$$

$$\dot{D} = \lambda (N - D) \tag{4.2}$$

$$\dot{T} = \gamma (N - 1 - bT) , \tag{4.3}$$

and

$$N(0) = 1 \qquad T(0) = 0$$

$$D(0) = 1 \qquad K_e(0) = 0 .$$

APPENDIX B: REACTOR STABILITY

As it was shown by Liapunov, "Problems general de la stabilite du mouvement," the general problem of stability of motion relative to certain quantities can be reduced in many cases to a solution of problems on stability of the trivial solution of the system. In theorem 1, "Direct Method," he stated: "If the system of differential equations that defines the system of differential equations that defines the system is such that it is possible to find a positive definite function V, the derivative of which, with respect to time, calculated by virtue of the system equations satisfies the inequality $dV/dt \leq 0$, then the system is stable."

The problem is to investigate whether or not the stationary state is stable with respect to initial disturbances.

The operation of the reactor (mathematical model) is defined by equations (3.1), (3.2) and (3.3), assuming that the external control is disconnected and neglecting the poisoning effects (since they represent a very slow phenomenon) the reactivity can be considered as some function of temperature, i.e.:

$$\delta K = - f(t).$$

First of all it should be noted that n and C are always positive (and so are N and D), since they cannot be negative for physical reasons. Now if the initial conditions are

$$n = n(t_o) \geq 0$$

$$C = C(t_o) \geq 0$$

$$T = T(t_o),$$

then the solutions n(t) and C(t) which correspond to these initial conditions satisfies the inequalities

$$n(t) \geq 0$$

$$C(t) \geq 0$$

so the stability will be investigated in the domain $D(n > 0, C > 0)$, inherent stability in the presence of a single group of delayed neutrons

$$\frac{dn}{dt} = -f(T)\, n - \frac{\beta}{\ell^*} n + \lambda C \qquad (5.1)$$

$$\frac{dC}{dt} = \frac{\beta}{\ell^*} n - \lambda C \qquad (5.2)$$

$$\frac{dt}{dt} = T(n-1) - \tau b T \qquad (5.3)$$

which obviously admit the special solution for the stationary state

$$T = T_1 \quad \text{some final temperature} \qquad (6.1)$$

$$n = n_o = 1 + bT \qquad (6.2)$$

$$C = C_o. \qquad (6.3)$$

Now if $\beta>0$, $\ell^*>0$, $\lambda>0$, $b>0$, $1+b=0$ are verified then this solution for the system is asymptotically stable; for physical reasons the first four inequalities of the statement

are always fulfilled and the fifth is a consequence of the fourth one.

Considering the Liapunov function

$$V = \int_{T_1}^{T} f(t)dt + \int_{n_o}^{n} \frac{T(n-n_o)}{n} dn + \int_{C_o}^{C} \frac{\tau(C-C_o)}{C} dc \quad (7)$$

in which the constants T_1, n_o and C_o are given by formulas (6.1), (6.2) and (6.3), the function V becomes zero for $T = T_1$, $n = n_o$ and $C = C_o$ and under the conditions given in Theorem (1) by M. V. Popov in [5] it is positive in the domain $n > 0$, $C > 0$.

From equations (5.1), (5.2) and (5.3) the total derivative of V with respect to time is

$$\frac{dV}{dt} = f(t)\ [r(n-1)-rbT] + r\frac{(n-n_o)}{n}\ [-f(T)n-\frac{\beta}{\ell^*} n + \lambda C]$$

$$+ r\ \frac{(C - C_o)}{C}\ [\ \frac{\beta}{\ell^*} n - \lambda C]$$

$$= -rbTf(T) - \frac{r}{nc}\ [\ \frac{\beta}{\ell^*} n - \lambda C][C(n-n_o)-n(C-C_o)]$$

$$= -rbTf(T) - \frac{rn_o}{nC}\ \ \frac{\beta}{\ell^*} n - \lambda C^{\ 2} \quad (8)$$

and again under the conditions of the same theorem, the function above is negative semi-definite within the range $n > 0$, $C > 0$ and become zero for

$$\frac{\beta}{\ell^*} n = \lambda C$$

where $T = T_1$.

Since dV/dt is negative semi-definite, it can be concluded that the solution is stable in the sense of Liapunov.

COMPUTER OUTPUT

The computer output shown here corresponds to different runs where failures in the XDS-9300 library hybrid executive package were detected.

```
ERROR IN P006        .20
ERROR IN P012        .15
ERROR IN P057        .15

INTERACTION BETWEEN T421 AND T420
INTERACTION BETWEEN T422 AND T420
INTERACTION BETWEEN T423 AND T420
INTERACTION BETWEEN T424 AND T420
INTERACTION BETWEEN T425 AND T420
INTERACTION BETWEEN T426 AND T420
INTERACTION BETWEEN T427 AND T420
INTERACTION BETWEEN T430 AND T420
INTERACTION BETWEEN T431 AND T420
INTERACTION BETWEEN T432 AND T420
INTERACTION BETWEEN T433 AND T420

TEST1
 A/D,D/A TRUNK LINES TEST
 PERCENTAGE ERRORS:

          .08398          -.02460
 VARIANCES

          .00000           .00000


TEST2
 INTEGRATORS RATE
 PERCENTAGE ERRORS:

 T(1)             1.00012
 T(2)              .00232


TEST3
 INITIAL CONDITIONS PERCENTAGE ERRORS:
 NEUTRON CONCENTRATION                 .05493
 DELAYED NEUTRON CONCENTRATION        2.00037
```

```
FINAL VALUE PERCENTAGE ERRORS:

  DELAYED NEUTRONS CONCENTRATION              .44188
  TEMPERATURE                                 .28460
  NEUTRONS CONCENTRATION                      .29163


TEST4
IDENTIFICATION PERCENTAGE ERRORS:
  PARAMETER B                               10.00000
  PARAMETER A                                2.50000
```

```
C     MAIN PROGRAM
      CALL ENABLE
C
      OUTPUT(101)' READ INSTRUCTIONS IN LINE PRINTER'
      WRITE(6,2)
      PAUSE
C
C     TEST POTSETTING AND DAC
      CALL TESTO
C
C     TEST INTEGRATORS I.C.:S AND A/D,D/A CONVERSION
      CALL TEST1
C
C     TEST INTEGRATORS RATE
      CALL TEST2
C
C     TEST PLANT INITIAL CONDICIONS AND FINAL VALUE
      CALL TEST3
C
C     TEST IDENTIFICATION SYSTEM
      CALL TEST4
C
C     IDENTIFY UNKNOWN PARAMETER B
      CALL RUN1
C
C     IDENTIFY UNKNOWN PARAMETER A
      CALL RUN2
C
    2 FORMAT(1H1,5X,'VERIFY THAT CLOCK IS IN RUN; DVM PATCH OFF;'/
     *12X,'ANALOG IN REAL TIME; DELAY F-F 11 IN .1 MS;'/
     *12X,'POT P411 IN ZERO'/1H1)
      END
```

```
      SUBRØUTINE TESTO
      DIMENSIØN IDAC(12)
      DIMENSIØN IPØT(16)
      DATA IDAC/4HT420,4HT421,4HT422,4HT423,4HT424,4HT425,
     *4HT426,4HT427,4HT430,4HT431,4HT432,4HT433/
      DATA IPØT/4HP001,4HP002,4HP003,4HP004,4HP005,4HP006,
     *4HP010,4HP011,4HP012,4HP015,4HP017,4HP021,4HP051,4HP053,
     *4HP055,4HP057/
      N=0
      CALL PØTSET
C
C     TEST PØTENTIØMETERS SETTING
C
      DØ 1 I=1,16
      CALL SETPØT(IPØT(I),.2000)
      CALL SCAN(IPØT(I),XY)
      IF(ABS(XY-.2).LE..0003)GØ TØ 1
      A=ABS(XY-.2)/.2*100.
      N=N+1
      WRITE(6,3)IPØT(I),A
    3 FØRMAT(5X,'ERRØR IN ',A4,5X,F5.2)
    1 CØNTINUE
C
C     TEST ACCURACY ØN DAC'S
C
      DØ 10  I=1,12
      CALL DAC(I,.2000)
      CALL SCAN(IDAC(I),XY)
      IF(ABS(XY-.2).LE..0003)GØ TØ 10
      A=ABS(XY-.2)/.2*100.
      WRITE(6,3)IDAC(I),A
      N=N+1
   10 CØNTINUE
      DØ 30 I=1,12
   30 CALL DAC(I,.2000)
```

```
C
C     TEST INTERACTIØN AMØNG DAC LINES
C
      DØ 25 I=1,12
      CALL DAC(I,0.0100)
      DØ 20 J=1,12
      IF(J.EQ.I)GØ TØ 20
      CALL SCAN(IDAC(I),XY)
      IF(ABS(XY-.2).LE..0003)GØ TØ 20
      N=N+1
      WRITE(6,21)IDAC(J),IDAC(I)
   21 FØRMAT(5X,'INTERACTIØN BETWEEN ',A4,' AND ',A4)
   20 CØNTINUE
      CALL DAC(I,0.2000)
   25 CØNTINUE
      IF(N.GT.0)ØUTPUT(101)' ERRØR IN TESTO'
      IF(N.GT.0)PAUSE
      RETURN
      END
```

```
      SUBROUTINE TEST1
C
C     TEST INTEGRATORS I.C.'S AND A/D,D/A CONVERSION
C
      CALL SETPOT(4HP004,0.5,4HP012,0.200)
      WRITE(6,3)
    3 FORMAT(/'  TEST1')
      CALL DAC(9,.0000,10,.0000,11,.0000)
      X1=X2=X12=X22=0.0
      CALL SETLINES(1,-1.,2,-1.)
C
C     LINE T000 CHANGES DPDT RELAYS
C     LINE T001 CHANGES THE SYSTEM TO RESET
C
      DO 1 I=1,1000
      CALL DAC(1,.4000,2,.4000)
S      LDA     =10000
      CALL DELAY
      CALL ADK(0,X1S,1,X2S)
      X1S=X1S/4.0;X2S=X2S/4.0
C     LINE T500 TEST THE OVERALL A/D,D/A CONVERSION
C     LINES T421 AND T501 TEST THE A001 I.C.
      CALL DAC(1,.0200,2,.0200)
      X1S=100.*X1S;X2S=100.*X2S
      X1=X1+X1S;X2=X2+X2S
      X12=X12+X1S*X1S;X22=X22+X2S*X2S
    1 CONTINUE
      X1M=X1/1000. ;X2M=X2/1000.
      X1M=10.*(10.-X1M);X2M=10.*(10.+X2M)
      X1S=X12/1000. -(X1/1000. )**2;X2S=X22/1000. -(X2/1000. )**2
      WRITE(6,2)X1M,X2M,X1S,X2S
    2 FORMAT(5X,'A/D,D/A TRUNK LINES TEST'/5X,'PERCENTAGE ERRORS'//5X,2F
     *15.5//5X,'VARIANCES'//5X,2F15.5//)
      RETURN
      END
```

```
      SUBROUTINE TEST2
C
C     TEST INTEGRATORS RATE
C
      DIMENSION T(2)
      WRITE(6,1)
    1 FORMAT(/'  TEST2'/5X,'INTEGRATORS RATE'/5X,'PERCENTAGE ERROR'/)
      CALL DAC(1,.0,2,.0,3,-.1,4,.0,5,.0,6,.0,7,.0,8,.0)
S      LDA     =10
      CALL DELAY
      CALL SETLINES(1,-1.0,2,1.0)
      CALL DAC(12,0.100)
      CALL SETPOT(4HP005,0.000,4HP002,0.000)
      CALL RESET(100)
      CALL COMPUTE
C
C     COMPUTES FOR 10 SECONDS
C
      CALL SETLINES(3,-1.0)
    5 IF(TEST(1).LT.0)GO TO 5
      CALL HOLD
C
C     READ INTEGRATORS VOLTAGES
C
      CALL ADK(1,T(1),2,T(2))
C
C     COMPUTE PERCENTAGE ERRORS AND PRINT MESSAGE
C
      DO 3 I=1,2
      T(I)=1.0+T(I)
    3 WRITE(6,2)I,T(I)
    2 FORMAT(5X,'T(',I1,')',5X,F15.5)
      CALL SETLINES(1,1.0,2,1.0,3,1.0)
      RETURN
      END
```

```
      SUBROUTINE TEST3
C
C     TEST PROBLEM INITIAL CONDITIONS AND FINAL VALUE
C
      COMMON /A/B1,B2,B3
      B1=B3=1.6249;B2=1.2498
      CALL SETPOT(4HP001,.2000,4HP002,.8000,4HP003,.5000,4HP004,.5000,4H
     *P005,.2000,4HP006,.4000,4HP010,.2000)
C
C     SET INITIAL CONDITIONS
C
      CALL DAC(1,.2000,2,.4000,3,-.4000,12,-.4000)
      CALL SETLINES(2,-1.0)
S      LDA     =100000
      CALL DELAY
C
C     READ INITIAL CONDITIONS AND COMPUTE PERCENTAGE ERRORS
C
      CALL ADK(4,X,1,Y)
      X=X/2.0
      Y=Y/4.0
      X=100.*(1.0-5.0*X);Y=10.*(.1-Y)
      WRITE(6,1)X,Y
C
C     COMPUTE FINAL VALUES AND COMPARE WITH THE
C     KNOWN NUMERICAL SOLUTION
C
      CALL SETLINES(2,1.0)
      CALL COMPUTE
      DO 5 I=1,2000
S      LDA     =1000
      CALL DELAY
    5 CONTINUE
      CALL HOLD
      CALL ADK(1,X1,2,X2,4,X3)
```

```
      X1=X1/4.0;X2=X2/4.0;X3=X3/2.0
      X1=-10.*X1;X2=-10.*X2;X3=10.*X3
      X1=ABS((B1-X1)/B1)*100.;X2=ABS((B2-X2)/B2)*100.;X3=ABS((B3-X3)/B3)
     **100.0
      WRITE(6,3)X1,X2,X3
      RETURN
C
    1 FØRMAT(/'  TEST3'/'  INITIAL CØNDITIØNS PERCENTAGE ERRØRS:'/5X,'NE
     *UTRØN CØNCENTRATIØN ', 9X,F15.5/5X,'DELAYED NEUTRØN CØNCENTRATIØN
     * ',F15.5/)
    3 FØRMAT(/'  FINAL VALUE PERCENTAGE ERRØRS:'/5X,'DELAYED NEUTRØNS CØ
     *NCENTRATIØN',1X,F15.5/5X,'TEMPERATURE ',19X,F15.5/5X,'NEUTRØNS CØN
     *CENTRATIØN',9X,F15.5/)
      END
```

```
      SUBROUTINE TEST4
C     TEST IDENTIFICATION SYSTEM
      COMMON /A/B1,B2,B3
      COMMON /B/ACONST
C     IDENTIFICATION OF THE UNKNOWN PARAMETER B USING A KNOWN NUMERICAL
C     SOLUTION FOR THE STEADY STATE
    5 CONTINUE
      ACONST=0.6
C
C     SET LINE 5 SWITCHES  R006  FOR THE DIFFERENT IDENTIFICATION
C
      CALL SETLINES(5,1.0)
      CALL SETPOT(4HP051,ACONST,4HP053,0.9090,4HP055,ACONST,4HP057,
     *ACONST)
      CALL SETPOT(4HP011,ACONST,4HP015,ACONST,4HP017,ACONST,4HP021,ACONS
     *T)
C
C     COMPUTE AND SET INPUT SIGNALS INTO THE IDENTIFIER
C
      TL23=-.01*ACONST*1.2498
      TL25=0.0
      TL26=.01*ACONST*10.*B2*B2
      TL27=.01*ACONST*B2*B3
      CALL DAC(4,TL23,6,TL25,7,TL26,8,TL27)
      CALL COMPUTE
      DO 1 I=1,2000
S      LDA     =1000
      CALL DELAY
    1 CONTINUE
      CALL HOLD
C
C     READ VALUE OF B
C
C
C     COMPUTE PERCENTAGE ERROR AND PRINT MESSAGE
```

~~Oscar Eduardo~~
~~Barreto Campos~~
~~Braina~~ - ~~Gerber~~

U.S. DEPARTMENT OF COMMERCE
National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

November 29, 1972

United States Naval Postgraduate School 72-5
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BO W. THOTT
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To: Mr. Thott

I regret that our Library copy is also incomplete in that it lacks page 75, too. We have contacted the advisor in a search for the original but it appears that it has either been discarded or was retained by the author, who is an officer in the Peruvian Navy and who has now returned to his country.

George R. Luckett

-George R. Luckett
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Request for Replacement NTIS-212 (11-72)

```
C     RESET RELAYS
      CALL DAC(8,.0,9,.0,10,.0,11,.0)
      CALL SETLINES(5,1.0)
      RETURN
      END
```

```
      SUBROUTINE RUN1
C
C     SUBROUTINE TO CONTROL THE IDENTIFICATION PROCESS
C     FOR THE PARAMETER B
C
      COMMON/C1/X,GAMM,ALAMB,B,DEL,PP,R,XP,XE
      DIMENSION X(3),PE(4),PP(4),XE(2),G(2),PHI(4),XP(2)
      CONNECT(53,SAMP1)
      TL20=0.1
      T=.2
      X2=0.0
      OUTPUT(101)' P411 CONTROLS NOISE LEVEL'
      OUTPUT(101)' P412 CAN BE USED TO CONTROL NOISE BIAS'
      PAUSE
C
C     POT'S ALREADY IN THE RIGHT VALUE (TEST3)
C
C     SET PLANT INITIAL CONDITIONS
C
      CALL DAC(1,.2000,2,.4000,3,-.4000,12,-.4000)
      CALL RESET(1000)
      CALL COMPUTE
C
C     ACTIVATE INTERRUPT
C
      CALL SETLINES(4,-1.0)
      I=5000
      TL23=TL25=TL26=TL27=0.0
    1 IF(I.NE.0)GO TO 1
C
C     RESET INTERRUPT
C
   2$ CALL SETLINES(4,1.0)
      TL20=0.1
      CALL SCAN(4HA013,B)
```

```
CALL HØLD
B=100.*B
ØUTPUT(101) B
RETURN
```

```
      SUBRØUTINE SAMP1
      CØMMØN /B/ACØNST
C
C     SUBRØUTINE TØ SAMPLE THE REQUIRED STATES
C     FØR THE REGRESSIØN EQUATIØN ØF PARAMETER B
C     IT ALSØ PERFØRMS THE REQUIRED MULTIPLICATIØNS
C     AND SETS THE INPUT SIGNAL INTØ THE IDENTIFIER
C
      I=I-1
      X2I=X2
      CALL ADK(2,X2,4,X3)
      X2=2.5*X2;X3=5.*X3
      X2DØT=(X2-X2I)/T
      TL23=.01*ACØNST*X2
      TL25=-.01*ACØNST*X2DØT*X2
      TL26= .01*ACØNST*X2*X2*10.
      TL27=-.01*ACØNST*X2*X3
      CALL DAC(4,TL23,6,TL25,7,TL26,8,TL27)
      IF(I.GT.4500)GØ TØ 1
      IF(I.LT.4500.AND.I.GT.4000)TL20=TL20+0.0002
    1 CØNTINUE
      CALL DAC(1,TL20)
      IF(I.EQ.0)GØ TØ 2$
      RETURN
      END
```

```
      SUBROUTINE RUN2
C
C     SUBROUTINE TO CONTROLTHE IDENTIFICATION PROCESS
C     FOR THE UNKNOWN PARAMETER A
C
      COMMON /C1/X,GAMM,ALAMB,B,DEL,PP,R,XP,XE
      DIMENSION X(3),PE(4),PP(4),XE(2),G(2),PHI(4),XP(2)
      DATA NT,EPS,T/0,0.1E-02,0.0/
      DATA GAMM,ALAMB/1.0,.4/
      CONNECT(53,HANDY)
      NAMELIST
      DEL=0.02
      ACONST=0.6
    3 CONTINUE
C
C     INITIALIZE ESTIMATOR
C
      PP(4)=PP(1)=10.;PP(2)=PP(3)=XE(1)=XE(2)=XP(2)=X(2)=0.0
      XP(1)=X(1)=1.0;X(2)=2.0
      AK=0.5
      TL20=0.4*AK
      T=.2
      R=0.1
C
C     ACTIVATE IDENTIFIER
C
      CALL SETLINES(5,-1.0)
C     X3(0)=-X(1)/(AK-A*X(2)-1.0)
C
C     SET PLANT INITIAL CONDITIONS
C
      CALL DAC(1,.2000,2,.4000,12,-.4000)
      CALL RESET(1000)
      I=5000
      CALL COMPUTE
```

```
C
C     ACTIVATE INTERRUPT
C
      CALL SETLINES(4,-1.0)
    1 IF(I.NE.0)GØ TØ 1
   3$ CALL SETLINES(4,1.0)
      CALL SCAN(4HA056,A)
      CALL HØLD
      A=100.*A
      ØUTPUT(101)A
      INPUT(101)
      IF(I.EQ.0)GØ TØ 3
      RETURN
```

```
      SUBRØUTINE HANDY
C
C     SUBRØUTINE TØ SAMPLE THE REQUIRED STATES
C     FØR THE REGRESSION EQUATIØN ØF PARAMETER A
C     IT ALSØ PERFØRMS THE REQUIRED MULTIPLICATIØNS
C     AND SETS THE INPUT SIGNAL INTØ THE IDENTIFIER
C
      I=I-1
      CALL ADK(2,X(2),4,X(3))
C     NØRMALIZE
      X(2)=-2.5*X(2)
      X(3)=5.*X(3)
      CALL ESTIM
      TL27=-.01*X(3)*X(3)*XE(2)*ACØNST
      TL30= XE(2)*XE(2)*X(3)*X(3)*0.1*ACØNST
      TL31= .01*AK*XE(2)*X(3)*X(3)*ACØNST
      TL32= .01*XE(1)*XE(2)*X(3)*ACØNST
      CALL DAC(8,TL27,9,TL30,10,TL31,11,TL32)
      CALL SETLINES(7,-1.0)
      IF(I.EQ.0)GØ TØ 3$
      RETURN
      END
```

```
      SUBROUTINE ESTIM
C
C     KALMAN FILTER
C
C     SUBROUTINE TO ESTIMATE THE STATE VARIABLE D
C     IT GETS THE VALUE OF THE PARAMETER B FROM RUN1
C
      COMMON /C1/X,GAMM,ALAMB,B,DEL,PP,R,XP,XE
      DIMENSION X(3),PE(4),PP(4),XE(2),G(2),PHI(4),XP(2)
      X3=X(3)
C
C     COMPUTE GAINS
C
      G(1)=PP(3)/(PP(4)+R)
      G(2)=PP(4)/(PP(4)+R)
C
C     COMPUTE ESTIMATED VARIANCES
C
      PE(1)=PP(1)-G(1)*PP(2)
      PE(2)=(1.0-G(2))*PP(2)
      PE(3)=PP(3)-G(1)*PP(4)
      PE(4)=(1.0-G(2))*PP(4)
      PHI(1)=1.0-DEL*ALAMB
      PHI(2)=PHI(3)=0.0
      PHI(4)=1.0-DEL*B*GAMM
C
C     COMPUTE PREDICTED VARIANCES
C
      PP(1)=PHI(1)*PHI(1)*PE(1)
      PP(2)=PHI(1)*PHI(4)*PE(2)
      PP(3)=PHI(1)*PHI(4)*PE(3)
      PP(4)=PHI(4)*PHI(4)*PE(4)
C
C     COMPUTE ESTIMATED VALUES OF X
C
```

```
      XE(1)=XP(1)+G(1)*(X(2)-XP(2))
      XE(2)=XP(2)+G(2)*(X(2)-XP(2))
C
C     COMPUTE PREDICTED VALUES OF X
C
      XP(1)=PHI(1)*XE(1)+DEL*ALAMB*X3
      XP(2)=PHI(4)*XE(2)+DEL*GAMM*X3-DEL*GAMM
      RETURN
      END
```

LIST OF REFERENCES

1. G. E. Atomic Power Equipment Department, San Jose, California, GEAP-4418 (1964). Liquid Metal Fast Breeder Reactor Design Study (1000 MWe UO_2-PUO_2 Fueled Plant) by M. J. McNelly.

2. Kerlin, T. W., "Stability Extreme in Nuclear Power System with Design Incertainties." Nuclear Science and Engineering 27, p. 120-130, 1967.

3. Potts, T. F., Orstein, G. N., Clymer, A. B., "The Automatic Determination of Human and Other System Parameters." Proceedings of the Western Joint Computer Conference, Los Angeles, California, May 9 - 11, 1961, p. 645-660, 1961.

4. Rubin, A. I., Drivan, S., Meisser, W. W., "Regression Analysis and Parameter Identification." Simulation, July 1967, p. 39-45, 1961.

5. Bekey, G. A., McGhee, R. B., Gradient Methods for the Optimization of Dynamic System Parameters by Hybrid Computation, Academic Press, 1964.

6. Roman, W. S., Hsu, Chun, Habegger, L. J., "Parameter Identification in a Non Linear Reactor System," IEEE Transactions on Nuclear Science, February 1971, No. 1, p. 418-425, 1971.

7. Bekey, G. A., Karplus, W. J., Hybrid Computation, John Wiley and Sons, 1968.

8. Wilkie, D. F., Perkins, W. R., "Generation of Sensitivity Functions for Linear Systems Using Low Order Models," IEEE Transactions on Automatic Control, April 1969, V. AC-14, No. 2, p. 123-129, 1969.

9. Bekey, G. A., "Optimization of Multiparameter Systems by Hybrid Computer Techniques," Part I and II, Simulation, February and March 1964, p. 19-32 and 21-29, 1964.

10. Meissinger, H. F., Bekey, G. A., "An Analysis of Continuous Parameter Identification Methods," Simulation, February 1966, p. 94-102, 1966.

11. Goldberger, A. S., Topics in Regression Analysis, Macmillan, 1968.

12. Huang, D. S., Regression and Econometric Methods, John Wiley and Sons, 1970

13. Otterman, J., "The Properties and Methods for Computation of Exponentially-Mapped-Past Statistical Variables," IRE Transactions on Automatic Control, January 1960, p. 11-17, 1960.

14. Lee, R. C. K., Optimal Estimation, Identification and Control, M.I.T. Press, Cambridge, 1964.

15. Demetry, J., Statistical Control, Class notes U.S.N.P.G.S., 1970.

16. Gerba, A., Jr., A Computer Application in Nuclear Reactor Controller Design, paper presented at the Joint Conference on Mathematical and Computer Aids to Design, 27 October 1969, Anaheim, California, 1969.

17. Schultz, Control of Nuclear Reactors and Power Plants, Mc-Graw Hill, 1961.

18. Lipinsky, W. C., Vancroux, A. G., "Optimal Digital Computer Control of Nuclear Reactors," IEEE Transactions on Nuclear Science, February 1970, v. 17, p. 510-516, 1970.

19. Gyftopulos, E. P.,"Point Reactor Kinetics and Stability Criteria," Third United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, p. 270-279, May 1964.

20. Richards, J. C., "Automatic Checkout of a Large Hybrid Computer System," Fall Joint Computer Conference, 1968, p. 131-143, 1968.

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13. ABSTRACT

A continuous identification of parameters is performed on a simulated fast breeder nuclear reactor system using hybrid computation and applying techniques of statistical regression analysis and exponentially-mapped-past functions. Output states which are not directly measurable are estimated by use of a Kalman filter.

The method developed in this study is applied to a numerical example which demonstrates that unknown parameters can be identified within 3% of their actual value, with signal noise ratios as low as 10:1 in the measured states. The example also demonstrates that convergence occurs in a reasonably short time.

An executive software routine was written in FORTRAN IV for the XDS-9300 digital computer and was applied both for the control of the process and for establishing the operational performance and accuracy of the Comcor Ci-5000 analog computer that simulated the nuclear reactor system.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Regression Analysis						
Parameter Identification						
Nuclear Reactor						

Thesis
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Solution of a nuclear reactor parameter identification problem.